

DISTRICT OF COLUMBIA

Mathematics Pre-K through Grade 12 Standards



MATHEMATICS

INTRODUCTION

As the District of Columbia rallies to ensure that all of our young people are prepared to succeed in high school, college or other postsecondary options, there is no more important priority than having strong academic standards.

These are the foundation of all our efforts to strengthen instruction. They tell students and parents what matters most. They provide the framework for lesson plans, reading lists, homework assignments and the other curriculum tools that guide teachers in their classrooms. They are the starting point for professional development for teachers, principals and other educators. And they drive our state assessment system.

Fortunately, the District of Columbia has some of the strongest and clearest mathematics standards in the country. They are the work of many DC teachers and administrators, early childhood providers, community stakeholders, parents and others in a process that was coordinated by StandardsWork, Inc. They were adapted from standards used in the Commonwealth of Massachusetts and a set of prekindergarten standards developed by a broad-based team working under the direction of the executive director of the District government's Early Care and Education Administration (formerly Office of Early Childhood Development).

We hope that you find these tools useful as you work to ensure that all of our students receive the education they deserve.

Deborah A. Gist

State Superintendent of Education

September 2007

The learning standards specify what students should know and be able to do as learners of mathematics at the end of each grade level or course. Students are held responsible for learning standards listed at earlier grades, as well as for their current grade.

Organization of the Document

This document is organized into four parts:

- A statement of guiding philosophies that articulate a set of beliefs about teaching, learning, and assessing mathematics in the District of Columbia, pp. 3–5
- Grade-by-grade standards, organized as described below, pp. 6–51
- A glossary that explains words and phrases found in the standards, pp. 52–57
- Acknowledgments, pp. 58–60

Each learning standard in every grade or course has a unique identifier that consists of:

- *Grade level*: Pre-K, K, 1, 2, 3, etc. or course
AI = Algebra I; G = Geometry; AII = Algebra II;
PS = Probability and Statistics; and
PCT = Precalculus and Trigonometry
- *Strand*: NSO = Number Sense and Operations;
PRA = Patterns, Relations, and Algebra; G =
Geometry; M = Measurement; and DASP =
Data Analysis, Statistics, and Probability
- *Substrand category*, such as Fractions,
Decimals and Percents (NSO-F) and Estimation
(NSO-E) within Number Sense and Operations.
- *Standard number*: 1, 2, 3, etc.

For example, standard 4.G.6 is the sixth standard of the Geometry strand in grade 4. Standard AI.N.12 is the twelfth standard of the Number Sense and Operations strand in the Algebra I course. This numbering system also allows teachers to organize the standards by grade. For example, 5th grade teachers preparing their curriculum can distinguish grade 5 standards in each strand by identifying all of the standards beginning with a 5.

The mathematics learning standards for prekindergarten through grade 8 are organized by grade level and presented in five strands:

Number Sense and Operations (pp. 6–17)

- Number Sense (NSO-N)
- Fractions, Decimals and Percents (NSO-F)
- Estimation (NSO-E)
- Computation and Operations (NSO-C)

Patterns, Relations, and Algebra (pp. 18–23)

Geometry (pp. 24–29)

Measurement (pp. 30–35)

Data Analysis, Statistics, and Probability (pp. 36–41)

The standards for grades 9 through 12 are organized differently. The mathematics studied in high school fall naturally under the following discipline headings:

- **Algebra I (pp. 42–43)**
- **Geometry (pp. 44–45)**
- **Algebra II (pp. 46–47)**
- **Probability and Statistics (pp. 48–49)**
- **Precalculus and Trigonometry (pp. 50–51)**

To allow schools and teachers flexibility, the standards do not mandate that a particular high school course be initiated and completed in a single grade. For example, students could take geometry in grade 9, 10, or 11, depending on the preferred sequence of course offerings at each high school, but all students are required to take Algebra I and Geometry in order to graduate. We stress that the content included in middle school through the high school courses of Algebra I and Geometry represent a minimum; they define what will be assessed and what will be required for graduation. Students must have the opportunity to learn significantly more, including the opportunity to study proper algebra in grade 8 — if not in grade 7 — as do students in Singapore and Japan.

Although the District of Columbia presents standards for only two post-Algebra II courses — Probability and Statistics and Precalculus — there are other mathematics courses that

schools might offer concurrent with or subsequent to pre-calculus. Among these options are discrete mathematics and calculus. Schools also should provide interested students with enrichment options in mathematics such as Advanced Placement courses, independent research, internships, or opportunities to study special topics.

GUIDING PHILOSOPHIES FOR THE TEACHING AND LEARNING OF MATHEMATICS*

The following grade-specific standards envision all students in the District of Columbia achieving mathematical competence through a strong mathematics program that emphasizes problem solving, communicating, reasoning and proof, making connections, and using representations. These skills must be woven throughout the five strands of mathematics: Number Sense and Operations; Patterns, Relationships, and Algebra; Geometry; Measurement; and Data Analysis and Probability.

An effective mathematics program focuses on problem solving and requires teachers who have a deep knowledge of mathematics as a discipline.

These standards emphasize computational and procedural skills, conceptual understanding, and problem solving. These three components are not distinct from each other;

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rather, they are related and mutually reinforcing. Problem solving challenges students to apply their knowledge of concepts, to exercise their computational and procedural skills, and to use mathematics as a way to find answers to new or complex problems. To become able problem solvers, students need extensive opportunities to formulate questions, model problem situations in a variety of ways, generalize mathematical relationships, and solve problems at various levels of difficulty and at every level in their mathematical development. Problem solving is not separate from content; rather, students learn skills and concepts in order to apply them to solve problems in and outside of school. The elements of problem solving are consistent across grade levels. Students need to be able to distinguish relevant from irrelevant information, identify missing information, test ideas, try different approaches, explain their reasoning, check their results for errors and reasonableness of solutions, and devise independent ways to verify results. As students progress grade to grade, they should deal with problems that require increasingly more advanced knowledge, progressively more complex applications, and increased use of reasoning. They also should have problems that relate to the mathematics they are studying that year.

The study of mathematics is an exercise in reasoning that must go beyond acquiring procedural mathematical skills with their clear methods and boundaries. Students need to master the more subjective skills of reading, interpreting, representing, and communicating a problem and its solution. From the early grades on, students develop their reasoning skills by making and testing mathematical conjectures, drawing logical conclusions, and justifying their thinking in developmentally appropriate ways. In the early grades, for example, repeated addition becomes multiplication, multiplication of numbers less than 10 can be extended to numbers less than 100 and then to the entire number system, and knowing how to find the area of a right triangle extends to all right triangles. As they advance through the grades, students' arguments become more sophisticated, and they are able to use inductive and deductive reasoning to arrive at valid conclusions and construct proofs. By doing so, students learn what mathematical reasoning entails.

The ability to express mathematical ideas coherently to different audiences is an important skill in a technological society. Students develop this skill and deepen their understanding of mathematics when they use accurate mathematical language to talk and write about what they are doing. They

clarify mathematical ideas as they discuss them with peers and reflect on strategies and solutions. By talking and writing about mathematics and using the special symbols of mathematics correctly and precisely, students learn how to make convincing arguments and represent mathematical ideas verbally, pictorially, and symbolically. When they can do so, they have a set of tools that significantly expands their capacity to think mathematically.

Mathematics is not a collection of separate strands or standards. Rather, it is an integrated field of study. Students develop a perspective of the mathematics field as an integrated whole by understanding connections within and outside the discipline. This includes the ability to shift regularly between the specific and general (i.e., use specific examples to understand general ideas; extend specific results to more general cases). Major emphasis should be given to ideas and concepts across mathematical content areas that help students see that mathematics is a web of closely connected ideas. It also is important for teachers to demonstrate the significance and relevance of the subject by encouraging students to explore the connections that exist within mathematics, with other disciplines, and between mathematics and students' own experiences.

**Adapted from the Massachusetts Mathematics Curriculum Framework, November 2000.*

Technology has a role in a mathematics education.

Technology — in particular calculators and computers — can help promote students' understanding of mathematical concepts, quantitative reasoning, and achievement when used as a tool for solving problems, testing conjectures, accessing data, and verifying solutions. When properly used, technology can help students develop the skills and insight necessary to meet rigorous content standards in mathematics and prepare for the world beyond school. *However, appropriate use of calculators is essential;* technology cannot be used effectively without a student first understanding the mathematical skills, concepts, and relationships contained within the standard. Elementary students must learn how to perform thoroughly the basic arithmetic operations independent of the use of a calculator. Although the use of a graphing calculator can help middle and secondary students visualize properties of functions and their graphs, graphing calculators should be used only to enhance their understanding and skills rather than replace them. Well-designed computer programs can not only facilitate dynamic modeling of mathematical concepts, but they also allow students at different achievement levels to progress through different challenge levels as they master the content.

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STRAND: NUMBER SENSE AND OPERATIONS

Prekindergarten

NUMBER SENSE

PK.NSO.1. Use one-to-one correspondence (e.g., sees 4 children at table and gives each child 1 cup. Touches each doll as she counts how many are in the cradle.).

PK.NSO.2. Count with understanding to at least 10 (e.g., counts 10 blocks, pointing to each as he counts and then says, "I have 10!" Chooses and counts 7 beads to put on necklace.).

PK.NSO.3. Use numbers to tell how many (number quantity) (e.g., says, "I broke my cookie into 4 pieces." Takes attendance and says, "There are 10 boys and 9 girls.").

PK.NSO.4. Use numbers and counting as a means to solve problems, predict, and measure quantities (e.g., says, "5 cups" when asked to predict how many cups it will take to fill the bucket. Says, "Only 4 kids can ride tricycles now because that's all there are.").

PK.NSO.5. Recognize and name numerals up to 10 (e.g., points to each number on the toy clock while counting aloud. Points to sign and says, "See, only 4 kids can be at the water table.").

PK.NSO.6. Quickly recognize quantity of small groups of objects up to 4 (e.g., sees 3 bear counters and says, "There are 3 of them" without having to count them. While getting ready to paint at the easel, asks, "Why are there only 3 paint colors today? We always have 4!").

PK.NSO.7. Construct sets of a given number using concrete objects (e.g., counts 6 blocks to match the numeral 6. Plays a game of dominoes with a friend, lining up sides with the same number of dots to each other.).

PK.NSO.8. Use ordinal numbers and positional words in everyday activities (e.g., looks at picture schedule and describes what comes first, second, and third. Arranges objects in order [seriate] from small to large).

Kindergarten

K.NSO-N.1. Count by ones to at least 20.

K.NSO-N.2. Represent, name, and order a set of objects (up to 20).

K.NSO-N.3. Match quantities up to at least 10 with numerals and words.

K.NSO-N.4. Compare sets of up to at least 10 concrete objects using appropriate language (e.g., none, more than, fewer than, same number of, one more than).

K.NSO-N.5. Identify positions of objects in sequences (e.g., first, second) up to fifth.

K.NSO-N.6. Identify U.S. coins by name and determine their value.

FRACTIONS

K.NSO-F.7. Understand the concepts of whole and half.

NUMBER SENSE AND OPERATIONS

Patterns, Relations,
and Algebra

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 1

NUMBER SENSE

- 1.NSO-N.1.** Count, read, and write whole numbers to 110 and relate them to the quantities they represent (e.g., knows that 60 is bigger than 20).
- 1.NSO-N.2.** Compare and order whole numbers to 110 by using symbols for less than, equal to, or greater than ($<$, $=$, $>$).
- 1.NSO-N.3.** Identify the place value of the digits to 110.
- 1.NSO-N.4.** Represent equivalent forms of the same number through the use of physical models, diagrams, and number expressions (e.g., 9 may be represented as $4 + 5$, $3 + 6$, $3 + 3 + 3$, $10 - 1$, $12 - 3$).
- 1.NSO-N.5.** Identify numbers to 20 as odd or even.
- 1.NSO-N.6.** Make combinations of different coins up to 50 cents.

Grade 2

- 2.NSO-N.1.** Count, read, and write whole numbers to 1,000 and relate them to the quantities they represent.
- 2.NSO-N.2.** Compare and order numbers to 1,000; use the symbols $>$, $<$, $=$.
- 2.NSO-N.3.** Identify the place value of the digits to 1,000.
- 2.NSO-N.4.** Use words, models, and expanded forms (e.g., $35 = 3 \text{ tens} + 5 \text{ ones}$) to represent numbers to 1,000.
- 2.NSO-N.5.** Know that even numbers end in 0, 2, 4, 6, or 8; recognize even numbers as multiples of two; know that odd numbers end in 1, 3, 5, 7, or 9.
- 2.NSO-N.6.** Identify the value of all U.S. coins and \$1, \$5, \$10, and \$20 bills. Find the value of a collection of coins and dollar bills and different ways to represent an amount of money up to \$5.

FRACTIONS

- 1.NSO-F.7.** Model, identify, and represent fractions such as $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ as parts of wholes (e.g., $\frac{1}{4}$ of a pie) and parts of groups.
- 2.NSO-F.7.** Know that fractions may represent a portion of a whole that has been partitioned into parts of equal area or length; use the terms "numerator" and "denominator."
- 2.NSO-F.8.** Recognize the inverse relationship between the size of a unit fraction and the size of the denominator (e.g., $\frac{1}{4} < \frac{1}{3}$).
- 2.NSO-F.9.** Recognize, name, and write commonly used fractions such as $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.
- 2.NSO-F.10.** Recognize that fractions such as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{10}{10}$, and $\frac{100}{100}$ are equal to the whole and to one.

MATHEMATICS**STRAND: NUMBER SENSE AND OPERATIONS, CONTINUED****Prekindergarten****Kindergarten****COMPUTATION AND OPERATIONS**

PK.NSO.9. Demonstrate the idea of adding and subtracting by using concrete objects (e.g., while playing "Bears in a Cave," says, "I see 2 bears, so 1 must be hiding." Arranges 3 teddy bear counters in a block construction and then gets 1 more, saying, "Now I have 4.").

K.NSO-C.8. Use objects and drawings to model and solve related addition and subtraction problems to 10.

ESTIMATION

K.NSO-E.9. Estimate the number of objects in a group and verify results.

NUMBER SENSE AND OPERATIONS

Patterns, Relations,
and Algebra

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 1

COMPUTATION AND OPERATIONS

1.NSO-C.8. Demonstrate the ability to use conventional algorithms for addition and subtraction (two two-digit whole numbers).

1.NSO-C.9. Demonstrate an understanding of various meanings of addition and subtraction, such as addition as combination (i.e., plus, combined with, more), subtraction as comparison (i.e., how much less, how much more), equalizing (i.e., how many more are needed to make these equal), and separation (i.e., how much remaining).

1.NSO-C.10. Know addition and subtraction facts (addends to 10), commit to memory, and use them to solve problems.

1.NSO-C.11. Demonstrate the ability to fluently add and subtract one- and two-digit whole numbers that do not require regrouping.

1.NSO-C.12. Use mental arithmetic to find the sum or difference of two one-digit whole numbers.

1.NSO-C.13. Find the sum of three one-digit whole numbers (e.g., $3 + 4 + 2 =$).

1.NSO-C.14. Identify one more than, one less than, 10 more than, and 10 less than for any number up to 100.

1.NSO-C.15. Understand and use the inverse relationship between addition and subtraction (e.g., $8 + 6 = 14$ is equivalent to $14 - 6 = 8$ and is also equivalent to $14 - 8 = 6$) to solve problems and check solutions.

1.NSO-C.16. Know the meaning of "two times something" or "three times something" as an addition (e.g., two times seven means $7 + 7$).

Grade 2

2.NSO-C.11. Demonstrate the ability to use conventional algorithms for addition (two three-digit whole numbers and three two-digit whole numbers) and subtraction (two three-digit whole numbers).

2.NSO-C.12. Find the distance between numbers on the number line (e.g., how far is 76 from 24).

2.NSO-C.13. Know addition and subtraction facts (addends to 12), commit to memory, and use them to solve problems. Select and use appropriate operations (addition and subtraction) to solve problems, including those involving money.

2.NSO-C.14. Demonstrate the ability to add and subtract three-digit whole numbers accurately and efficiently.

2.NSO-C.15. Use mental arithmetic to find the sum or difference of two two-digit numbers.

2.NSO-C.16. Represent multiplication as repeated addition

2.NSO-C.17. Demonstrate proficiency with multiplication facts for the ones, twos, and fives.

2.NSO-C.18. Demonstrate an understanding of the inverse relationship of addition and subtraction, and use that understanding to simplify computation and check solutions.

2.NSO-C.19. Know and identify various meanings of addition and subtraction, such as addition as combination (i.e., plus, combined with, more), subtraction as comparison (i.e., how much less, how much more), equalizing (i.e., how many more are needed to make these equal), and separation (i.e., how much remaining).

ESTIMATION

2.NSO-E.20. Estimate, calculate, and solve problems involving addition and subtraction of two-digit numbers. Describe differences between estimates and actual calculations.

MATHEMATICS

STRAND: NUMBER SENSE AND OPERATIONS, CONTINUED

Grade 3

NUMBER SENSE

3.NSO-N.1. Exhibit an understanding of the base 10 number system by reading, modeling, and writing whole numbers to at least 10,000; demonstrate an understanding of the values of the digits.

3.NSO-N.2. Represent, compare, and order numbers to 10,000 using various forms, including expanded notation (e.g., $3,206 = 3 \times 1,000 + 2 \times 100 + 6$) and written out in words (e.g., three thousand two-hundred six).

3.NSO-N.3. Round whole numbers through 10,000 to the nearest 10, 100, and 1,000.

3.NSO-N.4. Recognize sets to which a number may belong (odd numbers, even numbers, and multiples of numbers through 10). Identify the numbers in those classes (e.g., the class of multiples of 7 between 1 and 29 consists of 7, 14, 21, 28).

Grade 4

4.NSO-N.1. Exhibit an understanding of the base 10 number system by reading, modeling, and writing whole numbers to at least 100,000; demonstrating an understanding of the values of the digits; and comparing and ordering the numbers.

4.NSO-N.2. Represent, compare, and order numbers to 100,000 using various forms, including expanded notation.

4.NSO-N.3. Round whole numbers to 100,000 to the nearest 10, 100, 1,000, 10,000, and 100,000.

4.NSO-N.4. Recognize sets to which a number may belong (odds, evens, multiples and factors of given numbers, and squares), and use these in the solution of problems.

4.NSO-N.5. Read and interpret whole numbers and decimals up to two decimal places; relate to money and place-value decomposition.

4.NSO-N.6. Determine if a whole number is a multiple of a given one-digit whole number and if a one-digit number is a factor of a given whole number.

4.NSO-N.7. Find all factors of a whole number up to 50; know that numbers such as 2, 3, 5, 7, and 11 do not have any factors except one and itself and that such numbers are called prime numbers.

4.NSO-N.8. Use concepts of negative numbers (e.g., on a number line, in counting, in temperature, in owing money).

FRACTIONS

3.NSO-F.5. Identify and represent fractions (between 0 and 1 with denominators through 10) as parts of unit wholes and parts of a collection.

3.NSO-F.6. Recognize, name, and use equivalent fractions with denominators 2, 3, 4, and 8; place these fractions on the number line; compare and order them and relate the number line to a ruler (e.g., $1/2 = 2/4 = 4/8$).

3.NSO-F.7. Know the meaning of 0.75, 0.50, and 0.25 as they relate to money; know that fractions and decimals are two different representations of the same concept (e.g., 50 cents is $1/2$ of a dollar, 75 cents is $3/4$ of a dollar).

3.NSO-F.8. Know that any fraction can be written as a sum of unit fractions (e.g., $3/4 = 1/4 + 1/4 + 1/4$).

3.NSO-F.9. Model and represent a mixed number (with denominator 2, 3, or 4) as a whole number and a fraction (e.g., $1\frac{2}{3}$, $3\frac{1}{2}$).

4.NSO-F.9. Demonstrate an understanding of fractions as parts of unit wholes, as parts of a collection, and as locations on a number line.

4.NSO-F.10. Know the relationships among halves, fourths, and eighths and among thirds, sixths, and twelfths; compare and order such fractions.

4.NSO-F.11. Recognize, name, and generate equivalent forms of common decimals (0.5, 0.25, 0.2, 0.1) and fractions (halves, quarters, fifths, and tenths) and explain why they are equivalent.

4.NSO-F.12. Select, use, and explain models to relate common fractions and mixed numbers (e.g., $1/2$, $1/3$, $1/4$, $1/5$, $1/8$, $1/10$, $1/12$, and $1\frac{1}{2}$); find equivalent fractions, mixed numbers, and decimals.

4.NSO-F.13. Represent positive decimals to the hundredths.

NUMBER SENSE AND OPERATIONS

Patterns, Relations,
and Algebra

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 5

NUMBER SENSE

5.NSO-N.1. Estimate, round, and manipulate very large (e.g., billions) and very small (e.g., thousandths) numbers; demonstrate an understanding of place value to billions and thousandths.

5.NSO-N.2. Represent and compare very large (billions) and very small (thousandths) positive numbers in various forms, such as expanded notation without exponents, e.g., $9,724 = (9 \times 1,000) + (7 \times 100) + (2 \times 10) + 4$.

5.NSO-N.3. Find and position integers, fractions, mixed numbers, and decimals (both positive and negative) on the number line.

5.NSO-N.4. Compare and order integers (including negative integers) and positive fractions, mixed numbers, decimals, and percents.

5.NSO-N.5. Apply the number theory concepts of common factor, common multiple, and divisibility rules for 2, 3, 5, and 10 to the solution of problems. Demonstrate an understanding of the concepts of prime and composite numbers.

5.NSO-N.6. Know the set of prime numbers to 100.

5.NSO-N.7. Determine the prime factors of all numbers through 100, and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

Grade 6

6.NSO-N.1. Explain the properties of and compute with rational numbers, expressed in a variety of forms.

6.NSO-N.2. Compare and order positive and negative fractions, decimals, and mixed numbers and place them on a number line.

6.NSO-N.3. Know that numbers and their opposites add to 0 and are on opposite sides and at equal distance from 0 on a number line; know that 0 is an integer that is neither negative nor positive.

6.NSO-N.4. Represent rational numbers as repeating or terminating decimals when possible, and translate between these representations.

6.NSO-N.5. Identify and determine common equivalent fractions, mixed numbers, decimals, and percentages.

6.NSO-N.6. Apply number theory concepts – including prime and composite numbers; prime factorization; greatest common factor; least common multiple; and divisibility rules for 2, 3, 4, 5, 6, 9, and 10 – to the solution of problems.

6.NSO-N.7. Round whole numbers and decimals to any given place.

FRACTIONS

5.NSO-F.8. Explain different interpretations of fractions as a ratio of whole numbers, as parts of unit wholes, as parts of a collection, as division of whole numbers by whole numbers, and as locations on the number line.

5.NSO-F.9. Interpret percents as parts out of 100, use % notation, and express a part of a whole as a percentage.

5.NSO-F.10. Identify and determine common equivalent fractions, mixed numbers (with denominators 2, 4, 5, and 10), decimals, and percents, and explain why they represent the same value.

5.NSO-F.11. Write improper fractions as mixed numbers, and know that a mixed number represents the number of "wholes" and the part of a whole remaining (e.g., $5/4 = 1 + 1/4 = 1 \frac{1}{4}$).

MATHEMATICS

STRAND: NUMBER SENSE AND OPERATIONS, CONTINUED

Grade 3

COMPUTATION AND OPERATIONS

3.NSO-C.10. Demonstrate an understanding of and the ability to use conventional algorithms for the addition and subtraction of up to five-digit whole numbers.

3.NSO-C.11. Add and subtract up to four-digit whole numbers accurately and efficiently.

3.NSO-C.12. Use concrete objects and visual models to add and subtract common fractions (halves, thirds, fourths, sixths, and eighths) with like denominators.

3.NSO-C.13. Solve problems involving addition and subtraction of money amounts in decimal notation.

3.NSO-C.14. Know multiplication is the result of counting the total number of objects in a set of equal groups (e.g., 3×5 gives the number of objects in 3 groups of 5 objects).

3.NSO-C.15. Know division (\div) as another way of expressing multiplication, i.e., that division is the inverse of multiplication (e.g., $2 \times 3 = 6$ can be rewritten as $6 \div 2 = 3$ or $6 \div 3 = 2$).

3.NSO-C.16. Know multiplication facts through 10×10 and related division facts (e.g., $9 \times 8 = 72$ and $72 \div 9 = 8$). Use these facts to solve related problems (e.g., 3×5 is related to 3×50).

3.NSO-C.17. Solve simple problems involving multiplication of multidigit whole numbers by one-digit numbers ($2,431 \times 2$).

3.NSO-C.18. Solve division problems in which a multidigit whole number is evenly divided by a one-digit number (e.g., $125 \div 5$).

3.NSO-C.19. Multiply up to two-digit whole numbers by a one-digit whole number accurately and efficiently.

3.NSO-C.20. Use the commutative (order) and identity properties of addition and multiplication on whole numbers in computations and problem situations (e.g., $3 + 4 + 7 = 3 + 7 + 4 = 10 + 4$).

3.NSO-C.21. Know and apply the special properties of 0 and 1 in multiplication.

3.NSO-C.22. Use multiplication and division fact families to understand the inverse relationship of these two operations and to compare and check results (e.g., because $3 \times 8 = 24$, we know that $24 \div 8 = 3$ or $24 \div 3 = 8$).

Grade 4

4.NSO-C.14. Demonstrate an understanding of and the ability to use conventional algorithms for the addition and subtraction of multidigit whole numbers.

4.NSO-C.15. Add and subtract up to five-digit numbers accurately and efficiently.

4.NSO-C.16. Use concrete objects and visual models to add and subtract fractions where the denominators are equal or when one denominator is a multiple of the other (denominators 2 through 12, and 100).

4.NSO-C.17. Select, use, and explain various meanings and models of multiplication and division of whole numbers. Understand and use the inverse relationship between the two operations.

4.NSO-C.18. Know multiplication facts through 12×12 and the inverse division facts. Use these facts to solve related multiplication problems and compute related problems.

4.NSO-C.19. Demonstrate understanding of and ability to use the conventional algorithms for multiplication of up to a three-digit whole number by a two-digit whole number. Multiply three-digit whole numbers by two-digit whole numbers accurately and efficiently.

4.NSO-C.20. Demonstrate understanding of and the ability to use the conventional algorithm for division of up to a three-digit whole number with a single-digit divisor (with or without remainders). Divide up to a three-digit whole number with a single-digit divisor accurately and efficiently. Interpret any remainders.

4.NSO-C.21. Multiply fractions by whole numbers, using repeated addition and area rectangular models.

4.NSO-C.22. Mentally calculate simple products and quotients up to a three-digit number by a one-digit number (e.g., 400×7 , or $320 \div 8$).

4.NSO-C.23. Multiply and divide money amounts in decimal notation by using whole-number multipliers and divisors.

4.NSO-C.24. Determine the unit cost when given the total cost and number of units.

4.NSO-C.25. Select and use appropriate operations (addition, subtraction, multiplication, and division) to solve problems, including those involving money.

4.NSO-C.26. Select, use, and explain the commutative, associative, and identity properties of operations on whole numbers in problem situations, e.g., $37 \times 46 = 46 \times 37$, $(5 \times 7) \times 2 = 5 \times (7 \times 2)$.

4.NSO-C.27. Use the relationship between multiplication and division to simplify computations and check results.

NUMBER SENSE AND OPERATIONS

Patterns, Relations,
and Algebra

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 5

COMPUTATION AND OPERATIONS

5.NSO-C.12. Add with negative integers, subtract positive integers from negative integers, and verify the reasonableness of the results.

5.NSO-C.13. Add and subtract fractions (including mixed numbers) with like and unlike denominators (of 2, 3, 4, 5 and 10), and express answers in the simplest form.

5.NSO-C.14. Add and subtract positive decimals.

5.NSO-C.15. Solve problems involving multiplication and division of any whole number.

5.NSO-C.16. Demonstrate proficiency with division, including division with positive decimals and long division with multidigit divisors.

5.NSO-C.17. Show an understanding of multiplication and division of fractions; multiply positive fractions with whole numbers.

5.NSO-C.18. Simplify fractions in cases when both the numerator and the denominator have 2, 3, 4, 5, or 10 as a common factor. Show that two fractions are or are not equivalent by reducing to simpler forms or by finding a common denominator (e.g., show how $10/15 = 14/21$).

5.NSO-C.19. Multiply positive decimals with whole numbers.

5.NSO-C.20. Demonstrate an understanding of and compute (positive integer) powers of 10 (e.g., 10^2 , 10^5); compute examples as repeated multiplication.

5.NSO-C.21. Know integer subtraction is the inverse of integer addition; use the number line to model addition and subtraction of integers and add and subtract integers, with the exception of subtracting negative integers.

5.NSO-C.22. Demonstrate an understanding of how parentheses affect expressions involving addition, subtraction, and multiplication, and use that understanding to solve problems — e.g., $3 \times (4 + 2) = 3 \times 6$.

Grade 6

6.NSO-C.8. Select and use appropriate operations to solve problems involving addition, subtraction, multiplication, division, and positive integer exponents with whole numbers and with positive fractions, mixed numbers, decimals, and percentages.

6.NSO-C.9. Know integer subtraction is the inverse of integer addition; use the number line to model addition and subtraction of integers and add and subtract integers.

6.NSO-C.10. Accurately and efficiently add, subtract, multiply, and divide (with multidigit divisors) whole numbers and positive decimals.

6.NSO-C.11. Use prime factorization to add and subtract fractions with like and unlike denominators.

6.NSO-C.12. Accurately and efficiently add, subtract, multiply, and divide positive fractions (including mixed numbers) with like and unlike denominators. Simplify fractions.

6.NSO-C.13. Calculate given percentages of quantities, and solve problems involving discounts at sales, interest earned, and tips.

6.NSO-C.14. Solve simple proportion problems using such methods as unit rate, scaling, finding equivalent fractions, and solving the proportion equation $a/b = c/d$.

6.NSO-C.15. Apply laws of exponents to multiply whole number powers with like bases.

6.NSO-C.16. Understand multiplication of a negative number by a positive integer as repeated addition.

6.NSO-C.17. Apply the Order of Operations for expressions involving addition, subtraction, multiplication, and division with grouping symbols.

MATHEMATICS

STRAND: NUMBER SENSE AND OPERATIONS, CONTINUED

Grade 3

ESTIMATION

3.NSO-E.23. Estimate the sum and difference of two numbers with three digits (sums up to 1,000) and judge reasonableness of estimates.

3.NSO-E.24. Understand and use the strategies of rounding and regrouping to estimate quantities, measures, and the results of whole-number computations (addition, subtraction, and multiplication) up to two-digit whole numbers and amounts of money to \$100 and to judge the reasonableness of answers.

Grade 4

4.NSO-E.28. Estimate and compute the sum or difference of whole numbers and positive decimals to two places.

4.NSO-E.29. Estimate the answers to calculations involving addition, subtraction, or multiplication; know when approximation or a rounded solution is appropriate and use it to check the reasonableness of answers.

4.NSO-E.30. Select and use a variety of strategies (e.g., front-end, rounding, and regrouping) to estimate quantities, measures, and the results of whole-number computations up to three-digit whole numbers and amounts of money to \$1,000 and to judge the reasonableness of answers.

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|-----------------------------|----------------------------------|----------|-------------|--|
| NUMBER SENSE AND OPERATIONS | Patterns, Relations, and Algebra | Geometry | Measurement | Data Analysis, Statistics, and Probability |
|-----------------------------|----------------------------------|----------|-------------|--|

Grade 5

Grade 6

ESTIMATION

5.NSO-E.23. Estimate sums and differences of whole numbers, positive fractions, and positive decimals. Estimate products of whole numbers and products of positive decimals with whole numbers. Use a variety of strategies and judge reasonableness of answers.

6.NSO-E.18. Estimate results of computations with whole numbers and with positive fractions, mixed numbers, decimals, and percentages. Determine reasonableness of estimates.

MATHEMATICS

STRAND: NUMBER SENSE AND OPERATIONS, CONTINUED

Grade 7

NUMBER SENSE

7.NSO-N.1. Compare, order, estimate, and translate among integers, fractions, mixed numbers (i.e., rational numbers), decimals, and percents.

7.NSO-N.2. Know that in decimal form, rational numbers either terminate or eventually repeat; locate rational numbers on the number line; convert between common repeating decimals and fractions.

7.NSO-N.3. Know the concept of absolute value (e.g., $|-3| = |3| = 3$).

7.NSO-N.4. Represent numbers in scientific notation (positive powers of 10 only), and use that notation in problem situations.

7.NSO-N.5. Differentiate between rational and irrational numbers (i.e., know that irrational numbers cannot be expressed as the quotient of two integers and cannot be represented by terminating or repeating decimals).

7.NSO-N.6. Interpret positive whole-number powers as repeated multiplication and negative powers as repeated division or multiplication by the multiplicative inverse. Simplify and evaluate expressions that include exponents.

7.NSO-N.7. Apply number theory concepts, including prime factorization and relatively prime numbers, to the solution of problems (e.g., find the prime factorization of whole numbers, and write the results using exponents: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).

7.NSO-N.8. Express ratios in several ways (e.g., 3 cups to 5 people; 3:5; $3/5$); recognize and find equivalent ratios.

7.NSO-N.9. Know the meaning of a square root of a number and its connection to the square whose area is the number.

Grade 8

8.NSO-N.1. Explain the properties of and compute with real numbers expressed in a variety of forms.

8.NSO-N.2. Know that every rational number is either a terminating or repeating decimal and that every irrational number is a nonrepeating decimal.

8.NSO-N.3. Know that the absolute value is the distance of the number from 0; determine the absolute value and additive inverse of real numbers; determine the absolute value of rational numbers.

8.NSO-N.4. Read, write, and compare rational numbers in scientific notation (positive and negative powers of 10), and use them in calculations and problem situations.

8.NSO-N.5. Define, compare, order, and apply frequently used irrational numbers, such as $\sqrt{2}$ and π (e.g., show that if π is known to be irrational, then 3π and $\pi/3$ also are irrational).

8.NSO-N.6. Use the laws of exponents for integer exponents (e.g., write $2^2 \times 2^3$ as $2 \times 2 \times \dots$ and then as a single power of 2; write 2^{-3} as a fraction).

8.NSO-N.7. Demonstrate an understanding of the properties of arithmetic operations on rational numbers.

NUMBER SENSE AND OPERATIONS

Patterns, Relations,
and Algebra

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 7

Grade 8

COMPUTATION AND OPERATIONS

7.NSO-C.10. Compute with fractions (including simplification of fractions), integers, decimals, and percentages (including those greater than 100 and less than 1) using the four operations and combinations of the four operations.

7.NSO-C.11. Demonstrate an understanding of the properties of arithmetic operations on rational numbers (integers, fractions, and terminating decimals); convert terminating decimals into reduced fractions.

7.NSO-C.12. Select and use appropriate operations — addition, subtraction, multiplication, division — to solve problems with rational numbers and negative integers.

7.NSO-C.13. Calculate the percentage increase and decrease of a quantity.

7.NSO-C.14. Use ratios and proportions in the solution of problems involving unit rates, scale drawings, and reading of maps.

7.NSO-C.15. Take positive and negative rational numbers to positive whole number powers.

7.NSO-C.16. Apply the laws of exponents to multiply whole number positive and negative powers of whole numbers; divide whole number powers with like bases; explain the inverse relationship between negative and positive exponents.

7.NSO-C.17. Use the inverse relationships of addition/subtraction and multiplication/division to simplify computations and solve problems (e.g., multiplying by $\frac{1}{2}$ or 0.5 is the same as dividing by 2).

7.NSO-C.18. Use the associative, commutative, and distributive properties; properties of the identity and inverse elements (e.g., $-7 + 7 = 0$; $\frac{3}{4} \times \frac{4}{3} = 1$).

7.NSO-C.19. Know and apply the Order of Operations rules to expressions involving powers and roots.

8.NSO-C.8. Calculate weighted averages such as course grades, consumer price indexes, and sports ratings.

8.NSO-C.9. Solve problems involving ratio units such as miles per hour, dollars per pound, or persons per square mile.

8.NSO-C.10. Solve problems involving derived quantities such as density, velocity, and weighted averages.

8.NSO-C.11. Solve problems that involve markups, commissions, profits, and simple and compound interest.

8.NSO-C.12. Apply the rules of powers and roots to the solution of problems.

8.NSO-C.13. Use the inverse relationship between squaring and finding the square root of a perfect square integer to solve problems.

8.NSO-C.14. Multiply and divide numbers written in scientific notation.

8.NSO-C.15. Select and use appropriate operations — addition, subtraction, multiplication, division, and positive integer exponents — to solve problems with rational numbers, including negative rationals.

ESTIMATION

7.NSO-E.20. Estimate results of computations with rational numbers; determine estimates to a certain stated accuracy.

8.NSO-E.16. Estimate and solve problems with square roots; find square roots of perfect squares and approximate the square roots of nonperfect squares by locating them between consecutive integers.

8.NSO-E.17. Determine estimates to a certain stated accuracy.

MATHEMATICS

STRAND: PATTERNS, RELATIONS, AND ALGEBRA

Prekindergarten

PK.PRA.1. Sort and classify objects by more than one attribute — color, shape, size, number, etc. (e.g., sorts play dough cookies by size, color, or shape. Sorts a collection of buttons into those with 1-4 holes).

PK.PRA.2. Recognize, describe, and copy simple patterns (e.g., joins the teacher in a clapping pattern, *slap the knees, slap the knees, clap hands; slap the knees, slap the knees, clap hands*. Uses a stamp to repeat a pattern).

Kindergarten

K.PRA.1. Identify the attributes of objects as a foundation for sorting and classifying (e.g., a red truck, a red block, and a red ball share the attribute of being red; a square block, a square cracker, and a square book share the attribute of being square).

K.PRA.2. Sort and classify objects by attributes such as color, shape, size, number, and other properties and explain; identify objects that do not belong to a particular group (e.g., all these objects are red; those are green).

K.PRA.3. Identify, reproduce, describe, extend, and create color, rhythmic, shape, number, and letter repeating patterns with simple attributes.

K.PRA.4. Count by fives and tens up to at least 50.

Number Sense and Operations

PATTERNS, RELATIONS,
AND ALGEBRA

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 1

- 1.PRA.1.** Identify, reproduce, describe, extend, and create simple rhythmic, shape, size, number, color, and letter repeating patterns.
- 1.PRA.2.** Describe and create arithmetic progressions (e.g., 1, 4, 7, 10 . . . or 25, 23, 21 . . .).
- 1.PRA.3.** Identify arithmetic progressions on the hundreds chart.
- 1.PRA.4.** Skip count forward and backward by twos, fives, and tens up to at least 50, starting at any number and using appropriate aids (e.g., hundreds chart, number line).
- 1.PRA.5.** Write and solve number sentences from problem situations that express relationships involving addition and subtraction, including $+$, $-$, $<$, $>$, $=$.
- 1.PRA.6.** Apply knowledge of fact families to solve simple open sentences for addition and subtraction that have variables (e.g., $\square + 2 = 7$ and $10 - \square = 6$).

Grade 2

- 2.PRA.1.** Recognize and describe simple repeating and growing patterns using numbers, shapes, sizes, colors, and letters.
- 2.PRA.2.** Describe functions related to coin trades and measurement trades (e.g., 5 pennies make 1 nickel; 4 cups make 1 quart).
- 2.PRA.3.** Skip count forward and backward by twos, fives, and tens up to at least 100, starting at any number.
- 2.PRA.4.** Construct and solve open sentences with variables for addition and subtraction of up to two three-digit numbers (e.g., $42 + \square = 292$).
- 2.PRA.5.** Use the commutative and associative rules for addition to simplify mental calculations and to check results.

MATHEMATICS

STRAND: PATTERNS, RELATIONS, AND ALGEBRA, CONTINUED

Grade 3

3.PRA.1. Create, describe, and extend symbolic (geometric) patterns and addition and subtraction patterns.

3.PRA.2. Select appropriate operational and relational symbols to make an expression true (e.g., if $4 _ 3 = 12$, what operational symbol goes in the blank?).

3.PRA.3. Determine values of variables in simple equations involving addition, subtraction, or multiplication (e.g., $4106 - \nabla = 37$, $5 = \bigcirc + 3$, and $\square - \bigcirc = 3$).

3.PRA.4. Know and express the relationships among linear units of measure, i.e., unit conversions (e.g., 3 feet = 1 yard; 12 inches = 1 foot).

3.PRA.5. Extend and recognize a linear pattern by its rules (e.g., the number of legs on a given number of horses may be calculated by counting by fours or by multiplying the number of horses by 4).

Grade 4

4.PRA.1. Create, describe, extend, and explain geometric and numeric patterns, including multiplication patterns such as 3, 30, 300, and 3,000; generalize the rule for the pattern and make predictions when given a table of number pairs of a set of data.

4.PRA.2. Use letters and other symbols (e.g., \blacktriangle , x) as variables in expressions and in equations or inequalities (mathematical sentences that use $=$, $<$, and $>$).

4.PRA.3. Use pictures, models, tables, charts, graphs, words, number sentences, and mathematical notations to interpret mathematical relationships.

4.PRA.4. Solve problems involving proportional relationships, including unit pricing (e.g., 4 apples cost 80 cents, so 1 apple costs 20 cents) and map interpretation (e.g., 1 inch represents 5 miles, so 2 inches represent 10 miles).

4.PRA.5. Determine how change in one variable relates to a change in a second variable (e.g., input-output tables).

Number Sense and Operations

PATTERNS, RELATIONS,
AND ALGEBRA

Geometry

Measurement

Data Analysis, Statistics,
and Probability

Grade 5

5.PRA.1. Analyze and determine the rules for extending symbolic, arithmetic, and geometric patterns and progressions (e.g., ABBCCC ... ; 1, 5, 9, 13, ...; 3, 9, 27, ...).

5.PRA.2. Replace variables with given values, evaluate, and simplify (e.g., $2(\bigcirc) + 3$ when $\bigcirc = 4$).

5.PRA.3. Use the properties of equality to solve problems with whole numbers (e.g., if $x + 7 = 13$, then $x = 13 - 7$, therefore $x = 6$; if $3 \times \square = 15$, then $1/3 \times 3 \times \square = 1/3 \times 15$, therefore $\square = 5$).

5.PRA.4. Represent real situations and mathematical relationships with concrete models, tables, graphs, and rules in words and with symbols (e.g., input-output tables).

5.PRA.5. Interpret and evaluate mathematical expressions that use parentheses; use parentheses to indicate which operation to perform first when writing expressions containing more than two terms and different operations.

5.PRA.6. Solve problems involving proportional relationships using concrete models, tables, graphs, and paper-pencil methods.

5.PRA.7. Interpret graphs that represent the relationship between two variables in everyday situations.

Grade 6

6.PRA.1. Use the properties of equality to solve problems using letter name variables (e.g., $1/4 + x = 7/12$).

6.PRA.2. Write and solve one-step linear equations and check the answers.

6.PRA.3. Identify and describe relationships between two variables with a constant rate of change (e.g., perimeter-side relationship for a square, distance-time graphs, and conversions such as feet to inches). Contrast these with relationships where the rate of change is not constant.

6.PRA.4. Simplify expressions of the first degree by combining like terms, and evaluate using specific values.

6.PRA.5. Understand that adding or subtracting the same number to both sides of an equation creates a new equation that has the same truth values.

6.PRA.6. Understand that multiplying or dividing both sides of an equation by the same nonzero number creates a new equation that has the same truth values.

6.PRA.7. Distinguish between an algebraic expression and an equation.

6.PRA.8. Recognize when information given in a table, graph, or formula suggests a proportional or linear relationship.

6.PRA.9. Produce and interpret graphs that represent the relationship between two variables (x and y) in everyday situations.

MATHEMATICS

STRAND: PATTERNS, RELATIONS, AND ALGEBRA, CONTINUED

Grade 7

7.PRA.1. Extend, represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic expressions. Include arithmetic and geometric progressions (e.g., compounding).

7.PRA.2. Evaluate simple algebraic expressions for given variable values (e.g., $3a^2 - b$ for $a = 3$ and $b = 7$).

7.PRA.3. Use the correct order of operations to evaluate expressions (e.g., $3(2x) = 5$).

7.PRA.4. Create and use symbolic expressions for linear relationships, and relate them to verbal and graphical representations.

7.PRA.5. Use variables and appropriate operations to write an expression, equation, or inequality that represents a verbal description (e.g., 3 less than a number, $1/2$ as large as area A).

7.PRA.6. Write and solve two-step linear equations and check the answers.

7.PRA.7. Identify, describe, and analyze linear relationships between two variables. Compare positive rate of change (e.g., $y = 3x + 1$) to negative rate of change (e.g., $y = -3x + 1$).

7.PRA.8. Use linear equations to model and analyze problems involving proportional relationships.

7.PRA.9. Simplify numerical expressions by applying properties of rational numbers (e.g., identity, inverse) and operations of rational numbers (distributive, associative, commutative); justify the process used.

7.PRA.10. Use algebraic terminology including, but not limited to, variable, equation, term, coefficient, inequality, expression, and constant.

7.PRA.11. Plot the values of quantities whose ratios are always the same (e.g., cost to the number of an item, feet to inches, circumference to diameter of a circle). Fit a line to the plot and understand that the slope of the line equals the quantities.

Grade 8

8.PRA.1. Use tables and graphs to represent and compare linear growth patterns. In particular, compare rates of change and x - and y -intercepts of different linear patterns.

8.PRA.2. Set up and solve linear equations and inequalities with one or two variables using algebraic methods and graphs.

8.PRA.3. Use linear equations to model and analyze problems involving proportional relationships.

8.PRA.4. Identify the slope of a line as a measure of its steepness and as a constant rate of change from its table of values, equation, or graph. Apply the concept of slope to the solution of problems.

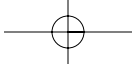
8.PRA.5. Identify the roles of variables within an equation (e.g., $y = mx + b$, expressing y as a function of x with parameters m and b).

8.PRA.6. Distinguish between numerical and algebraic expressions, equations, and inequalities.

8.PRA.7. Interpret the formula $(-x)(-y) = xy$ in calculations involving such things as distance, speed, and time, or in the graphing of linear functions. Use this identity to simplify algebraic expressions [e.g., $(-2)(-x+2) = 2x - 4$].

8.PRA.8. Explain and analyze - both quantitatively and qualitatively, using pictures, graphs, charts, and equations - how a change in one variable results in a change in another variable in functional relationships (e.g., $C = \pi d$, $A = \pi r^2$ (A as a function of r), $A_{\text{rectangle}} = lw$ ($A_{\text{rectangle}}$ as a function of l and w)).

8.PRA.9. Graph a linear equation using ordered pairs; identify and represent the graphs of linear functions.



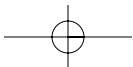
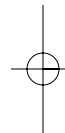
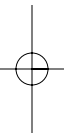
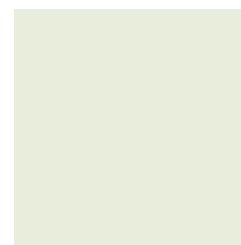
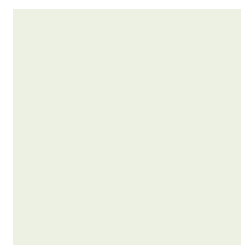
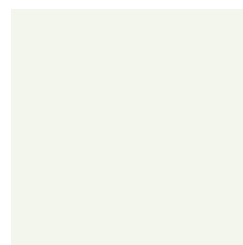
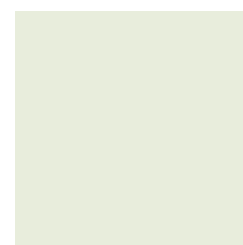
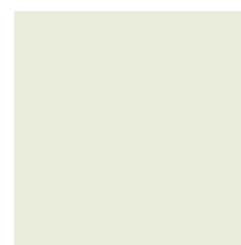
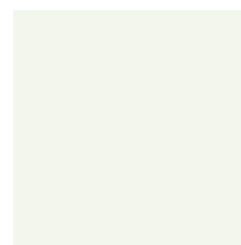
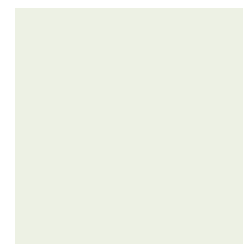
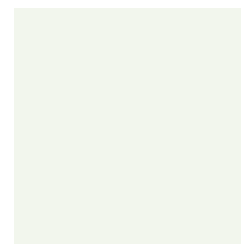
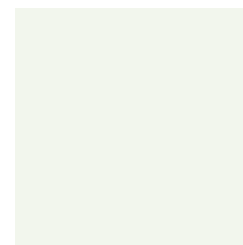
Number Sense and Operations

**PATTERNS, RELATIONS,
AND ALGEBRA**

Geometry

Measurement

Data Analysis, Statistics,
and Probability



MATHEMATICS

STRAND: GEOMETRY

Prekindergarten

PK.G.1. Describe, name, and interpret distance and position in space; understand and use positional words (e.g., turns Lotto game board so player sitting opposite can see it right side up. Frustrated, says, "I can't reach the ball; it's too high").

PK.G.2. Recognize, name, and describe simple two- and three-dimensional shapes (e.g., says, "This is a triangle. See, it has three sides." Says, "You need balls of clay to make a snowman.").

PK.G.3. Match, sort, and classify shapes (e.g., says, "These all go together because they have three sides." When cleaning up blocks, orders the different shapes on the shelf by matching them to the outlines on the shelf.).

PK.G.4. Put together and take apart shapes to make new shapes (e.g., makes a picture using a variety of pattern block shapes. Puts a straw across a square and says, "Now I have triangles.").

PK.G.5. Create shapes using concrete materials such as straws (e.g., uses toothpicks to make rectangles of different sizes. Puts a ball on top of a triangular block and says, "I'm eating an ice cream cone.").

Kindergarten

K.G.1. Name shapes of pattern blocks (e.g., triangle, square, circle).

K.G.2. Describe attributes of two-dimensional shapes (e.g., number of sides, number of corners, size, roundness); sort these shapes.

K.G.3. Identify and compare three-dimensional shapes (e.g., cube, box, sphere).

K.G.4. Identify positions of objects in space and use appropriate language (e.g., beside, inside, next to, close to, above, below, apart) to describe and compare their relative positions.

Number Sense and Operations

Patterns, Relations,
and Algebra

GEOMETRY

Measurement

Data Analysis, Statistics,
and Probability

Grade 1

- 1.G.1.** Describe attributes and parts of two- and three-dimensional shapes (e.g., length of sides and number of corners, edges, faces, and sides).
- 1.G.2.** Identify congruent shapes.
- 1.G.3.** Identify symmetry in two-dimensional shapes.
- 1.G.4.** Combine shapes and take them apart to make other shapes (e.g., two congruent right triangles can be arranged to form a rectangle).
- 1.G.5.** Arrange and describe objects in space by proximity, position, and direction (e.g., near, far, below, above, up, down, behind, in front of, next to, left or right of).

Grade 2

- 2.G.1.** Identify, describe, draw, and compare two-dimensional shapes, including both polygonal (up to six sides) and curved figures such as circles.
- 2.G.2.** Classify familiar two- and three-dimensional shapes by common attributes such as shape of curved and straight lines, number and shape of faces, edges, and vertices.
- 2.G.3.** Match and construct congruent (e.g., two triangles that are the same shape and size) and symmetric shapes (e.g., two halves of a heart divided down the center line).
- 2.G.4.** Identify shapes under rotation (turns), reflections (flips), translation (slides), and enlargement. Describe direction of translations (e.g., left, right, up, down).
- 2.G.5.** Predict and explain the results of putting two-dimensional shapes together and taking them apart (e.g., two congruent right triangular shapes form a rectangle).
- 2.G.6.** Relate geometric ideas to numbers (e.g., seeing rows in an array as a model of repeated addition).

MATHEMATICS

STRAND: GEOMETRY, CONTINUED

Grade 3

3.G.1. Compare and analyze attributes and other features (e.g., number and shape of sides, faces, corners, right angles) of two-dimensional geometric shapes, especially the attributes of triangles (isosceles, equilateral, right) and quadrilaterals (rectangle, square).

3.G.2. Describe, model, draw, compare, and classify three-dimensional and two-dimensional shapes, especially circles and polygons (e.g., triangles and quadrilaterals).

3.G.3. Identify angles as right, acute (less than a right angle), or obtuse (greater than a right angle).

3.G.4. Identify and draw lines that are parallel, perpendicular, and intersecting.

3.G.5. Identify and draw lines of symmetry in two-dimensional shapes.

3.G.6. Apply techniques such as reflections (flips), rotations (turns), and translations (slides) for determining if two shapes are congruent.

3.G.7. Using ordered pairs of whole numbers and/or letters, locate and identify points on a grid.

Grade 4

4.G.1. Compare and analyze attributes and other features (e.g., number of sides, faces, corners, right angles, diagonals, and symmetry) of two- and three-dimensional geometric shapes.

4.G.2. Describe, model, draw, compare, and classify two- and three-dimensional shapes (e.g., circles, polygons, parallelograms, trapezoids, cubes, spheres, pyramids, cones, cylinders).

4.G.3. Know the definitions of a right angle, an acute angle, and an obtuse angle. Understand that 90° , 180° , 270° , and 360° are associated, respectively, with $1/4$, $1/2$, $3/4$, and full turns.

4.G.4. Describe and draw intersecting, parallel, and perpendicular lines.

4.G.5. Recognize similar figures (two shapes, R and S, are similar if they are congruent after one of them is shrunk or expanded).

4.G.6. Describe and apply techniques such as reflections (flips), rotations (turns), and translations (slides) for determining if two shapes are congruent.

4.G.7. Predict and validate the results of partitioning, folding, and combining two- and three-dimensional shapes.

4.G.8. Using ordered pairs of numbers and/or letters, graph, locate, and identify points and describe paths (first quadrant).

Number Sense and Operations

Patterns, Relations,
and Algebra

GEOMETRY

Measurement

Data Analysis, Statistics,
and Probability

Grade 5

- 5.G.1.** Identify polygons based on their properties, including types of interior angles, perpendicular or parallel sides, and congruence of sides (e.g., squares, rectangles, rhombuses, parallelograms, and trapezoids; isosceles, equilateral, and right triangles).
- 5.G.2.** Identify, describe, and compare special types of three-dimensional shapes (e.g., cubes, prisms, spheres, cones, and pyramids) based on their properties, such as edges and faces.
- 5.G.3.** Identify relationships among points, lines, and planes (e.g., intersecting, parallel, perpendicular).
- 5.G.4.** Identify and describe types of symmetry, including line and rotational.
- 5.G.5.** Determine if two triangles or two quadrilaterals are congruent by measuring sides or a combination of sides and angles.
- 5.G.6.** Predict, describe, and perform transformations on two-dimensional shapes (e.g., translations, rotations, and reflections).
- 5.G.7.** Graph points and identify coordinates of points on the Cartesian coordinate plane in the first two quadrants.

Grade 6

- 6.G.1.** Match three-dimensional objects and their two-dimensional representations (e.g., nets, projections, and perspective drawings).
- 6.G.2.** Identify angles as vertical, adjacent, complementary, or supplementary; provide descriptions of these terms; and use the properties of complementary and supplementary angles and the sum of the angles of a triangle to solve problems involving an unknown angle.
- 6.G.3.** Determine if two shapes are congruent by motions or series of motions (e.g., translations, rotations, and reflections); predict the results of transformations on unmarked planes and draw the transformed figure (e.g., predict how tessellations transform under translation, reflection, and rotation).
- 6.G.4.** Graph points and identify coordinates of points on the Cartesian coordinate plane in all four quadrants.
- 6.G.5.** Find the distance between two points on horizontal or vertical number lines.

MATHEMATICS

STRAND: GEOMETRY, CONTINUED

Grade 7

7.G.1. Identify three-dimensional figures (e.g., prisms, pyramids) by their physical appearance, distinguishing attributes, and spatial relationships such as parallel faces.

7.G.2. Demonstrate an understanding of conditions that indicate two geometrical figures are congruent and what congruence means about the relationships between the sides and angles of the two figures.

7.G.3. Classify figures in terms of congruence and similarity, and apply these relationships to the solution of problems.

7.G.4. Know and understand the Pythagorean theorem and its converse. Apply the theorem to the solution of problems, including using it to find the length of the missing side of a right triangle, and perimeter, area, and volume problems.

7.G.5. Use compass, straightedge, and protractor to perform basic geometric constructions to draw polygons and circles.

7.G.6. Understand and use coordinate graphs to plot simple figures; determine lengths and areas related to them; and determine their image under translations, reflections, and rotations (e.g., predict how tessellations transform under translations, reflections, and rotations).

Grade 8

8.G.1. Analyze, apply, and explain the relationship between the number of sides and the sums of the interior and exterior angle measures of polygons.

8.G.2. Demonstrate an understanding of the relationships of angles formed by intersecting lines, including parallel lines cut by a transversal.

8.G.3. Demonstrate an understanding of conditions that indicate two triangles are similar: the corresponding angles are congruent (AAA similarity); the ratios of two pairs of corresponding sides are equal and the included angles are congruent (SAS similarity); ratios of all pairs of corresponding sides are equal (SSS similarity).

8.G.4. Use a straightedge, compass, protractor, or other tool to formulate and test conjectures and to draw geometric figures (Example: Draw the perpendicular bisector of a segment, an equilateral triangle, the bisector of an angle, diagonals, midpoints, radii, diameters, and chords of circles).

8.G.5. Apply spatial reasoning by recognizing and drawing two-dimensional representations of three-dimensional objects (e.g., nets, projections, and perspective drawings of cylinders, prisms, and cones).

8.G.6. Find the distance between two points on the coordinate plane using the distance formula; find the midpoint of the line segment; recognize that the distance formula is an application of the Pythagorean theorem.

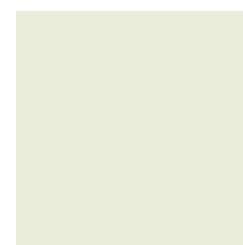
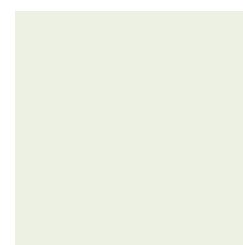
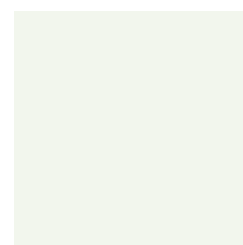
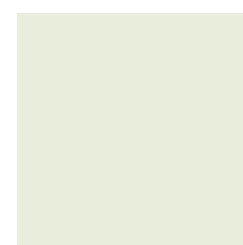
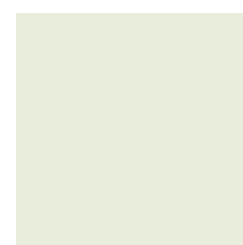
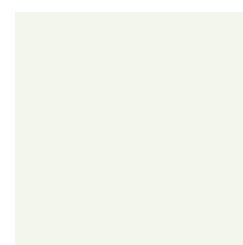
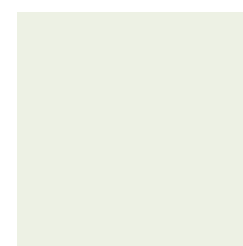
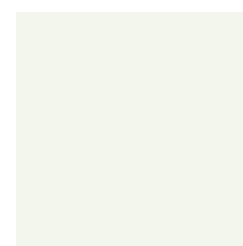
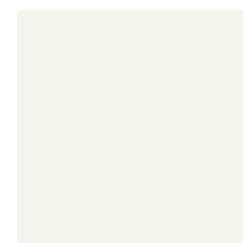
Number Sense and Operations

Patterns, Relations,
and Algebra

GEOMETRY

Measurement

Data Analysis, Statistics,
and Probability



MATHEMATICS**STRAND: MEASUREMENT****Prekindergarten**

PK.M.1. Identify appropriate tools of measurement (e.g., picks up a measuring cup and says, "We need to add 2 cups of water to the cake mix" in dramatic play. Experiments using a balance scale to see how many wooden cubes make one side go all the way down.).

PK.M.2. Make use of nonstandard and standard units for measurement to obtain information (e.g., uses footsteps to measure the length of the hopscotch grid on the playground. Looks at the clock and asks, "Is it time to go outside?").

PK.M.3. Show awareness of time concepts and sequence (e.g., says, "After lunch we have read-aloud time." Says, "We go home at 3 o'clock.").

Kindergarten

K.M.1. Recognize and compare objects with respect to the attributes of length, volume/capacity, weight, area, and time using appropriate language (e.g., longer, taller, shorter, same length; heavier, lighter, same weight; holds more, holds less, holds the same amount).

K.M.2. Make and use estimates of measurements from everyday experiences.

K.M.3. Use standard and nonstandard units to measure length.

K.M.4. Order events in a day.

K.M.5. Tell time to the nearest hour.

K.M.6. Identify U.S. coins and their value.

Number Sense and Operations

Patterns, Relations,
and Algebra

Geometry

MEASUREMENT

Data Analysis, Statistics,
and Probability

Grade 1

- 1.M.1.** Compare the length, weight, and volume of two or more objects by using direct comparison.
- 1.M.2.** Make and use estimates of measurement, including time and weight.
- 1.M.3.** Measure the length of objects by repeating a nonstandard or standard unit.
- 1.M.4.** Tell time at half-hour intervals on analog and digital clocks using a.m. and p.m., and relate time to events (e.g., before/after, shorter/longer).
- 1.M.5.** Make combinations of coins up to 50 cents.

Grade 2

- 2.M.1.** Measure and compare the length of common objects using metric and U.S. customary units to the nearest centimeter or inch.
- 2.M.2.** Make and use estimates of measurement including time, volume, weight, area, and perimeter.
- 2.M.3.** Select and correctly use the appropriate measurement tool (ruler, balance scale, thermometer).
- 2.M.4.** Tell time at quarter-hour intervals.
- 2.M.5.** Identify parts of the day (e.g., morning, afternoon, evening), days of the week, and months of the year. Identify dates using a calendar.
- 2.M.6.** Identify the value of all U.S. coins and \$1, \$5, \$10, and \$20 bills. Find the value of a collection of coins and bills and different ways to represent an amount of money up to \$5 using appropriate notation.

MATHEMATICS

STRAND: MEASUREMENT, CONTINUED

Grade 3

3.M.1. Demonstrate an understanding of such attributes as length, area, and weight; select the appropriate type of unit for measuring each attribute using both the U.S. customary and metric systems.

3.M.2. Carry out simple unit conversions within a system of measurement such as hours to minutes and cents to dollars (e.g., 1 hour = 60 minutes).

3.M.3. Identify time to the nearest 5 minutes on analog and digital clocks using a.m. and p.m. Compute elapsed time using a clock (e.g., hours and minutes since ...) and using a calendar (e.g., days since ...).

3.M.4. Estimate and find area and perimeter of a rectangle and triangle using diagrams, models, and grids or by measuring.

Grade 4

4.M.1. Identify and use appropriate metric and U.S. customary units and tools (e.g., ruler, protractor, graduated cylinder, thermometer) to estimate, measure, and solve problems involving length, area, volume, weight, time, angle size, and temperature.

4.M.2. Carry out simple unit conversions within a system of measurement (e.g., yards to feet or inches; gallons to quarts and pints).

4.M.3. Identify time to the minute on analog and digital clocks using a.m. and p.m. Compute elapsed time using a clock (e.g., hours and minutes since...) and using a calendar (e.g., days since ...).

4.M.4. Estimate and find area and perimeter of shapes, including irregular shapes, using diagrams, models, and grids or by measuring.

4.M.5. Recognize that rectangles that have the same area can have different perimeters; understand that rectangles that have the same perimeter can have different areas.

Number Sense and Operations

Patterns, Relations,
and Algebra

Geometry

MEASUREMENT

Data Analysis, Statistics,
and Probability

Grade 5

5.M.1. Apply the concepts of perimeter and area to the solution of problems involving triangles and rectangles. Apply formulas where appropriate.

5.M.2. Apply formulas for the areas of triangles, rectangles, and parallelograms; recognize that shapes with the same number of sides but different appearances can have the same area.

5.M.3. Solve problems involving proportional relationships and units of measurement.

5.M.4. Identify, measure, and describe circles and the relationships of the radius, diameter, circumference, and area (e.g., $d = 2r$), and use these concepts to solve problems.

5.M.5. Find volumes and surface areas of rectangular prisms.

5.M.6. Know that angles on a straight line add up to 180° , interior angles of a triangle add up to 180° , angles surrounding a point add up to 360° , and interior angles of a quadrilateral add up to 360° ; use these properties to solve problems.

5.M.7. Identify, measure, describe, classify, and draw various angles and triangles, given sides and the angle between them or given two angles and the side between them (e.g., draw a triangle with one right angle and two sides congruent).

Grade 6

6.M.1. Differentiate between and use appropriate units of measures for two- and three-dimensional objects (i.e., when finding perimeter, area, and volume).

6.M.2. Find areas of triangles and parallelograms. Recognize that shapes with the same number of sides but different appearances can have the same area.

6.M.3. Develop strategies to find the area and perimeter of complex shapes (e.g., subdividing them into basic shapes such as quadrilaterals, triangles, circles).

6.M.4. Solve problems involving proportional relationships and units of measurement (e.g., same system unit conversions, scale models, maps, and speed).

6.M.5. Understand the concept of volume; use the appropriate units in common measuring systems (e.g., cubic inch, cubic centimeter, cubic meter, cubic yard) to compute the volume of rectangular solids, including rectangular prisms.

6.M.6. Identify, measure, describe, classify, and construct various angles, triangles, and quadrilaterals; measure the interior angles of various polygons.

6.M.7. Understand the concept of the constant π ; know the formulas for the circumference and area of a circle. Use the concepts to solve problems.

6.M.8. Know and use the formulas for the volumes and surface areas of cubes and rectangular prisms, given the lengths of their sides.

6.M.9. Find the sum of the angles in simple polygons (up to eight sides) with and without measuring the angles.

MATHEMATICS

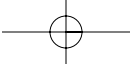
STRAND: MEASUREMENT, CONTINUED

Grade 7

- 7.M.1.** Select, convert (between systems of measurement), and use appropriate units of measurement or scale.
- 7.M.2.** Demonstrate an understanding of the concepts and apply formulas and procedures for determining measures, including those of area and perimeter/circumference of parallelograms, trapezoids, and circles. Given the formulas, determine the surface area and volume of rectangular prisms and cylinders.
- 7.M.3.** Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity; use models, graphs, and formulas to solve simple problems involving rates (e.g., velocity and density); check the units of the solutions; use dimensional analysis to check the reasonableness of the answer.
- 7.M.4.** Construct and read drawings and models made to scale.
- 7.M.5.** Use ratio and proportion, including scale factors, in the solution of problems.

Grade 8

- 8.M.1.** Given the formulas, convert from one system of measurement to another.
- 8.M.2.** Understand the concept of surface area and volume; given the formulas, determine the surface area and volume of rectangular prisms, cylinders, and spheres.
- 8.M.3.** Use a straightedge, compass, protractor, or other tools to formulate and test conjectures and to draw geometric figures.
- 8.M.4.** Solve problems about similar figures and scale drawings. Understand that when the lengths of all dimensions of an object are multiplied by a scale factor, the surface area is multiplied by the square of the scale factor and the volume is multiplied by the cube of the scale factor.
- 8.M.5.** Understand and use the fact that when two polygons or circles are similar with scale factor of r , their areas are related by a factor of r^2 .
- 8.M.6.** Use proportions to express relationships between corresponding parts of similar figures.



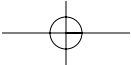
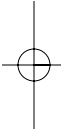
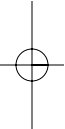
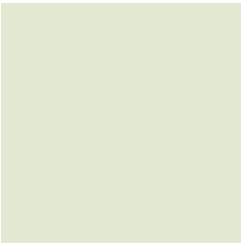
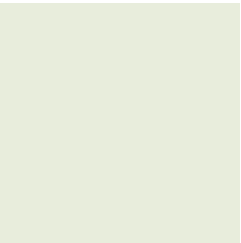
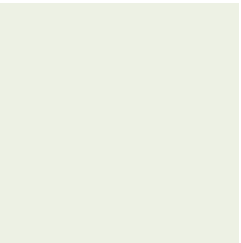
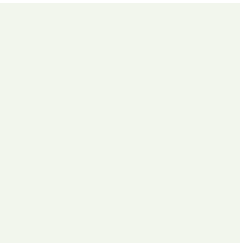
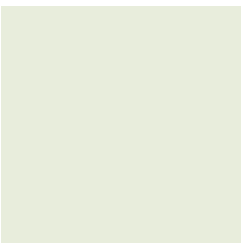
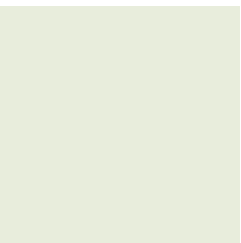
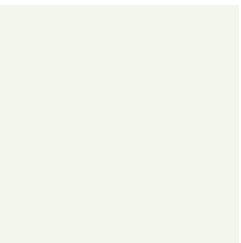
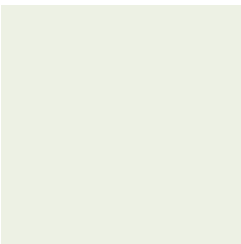
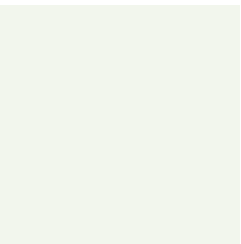
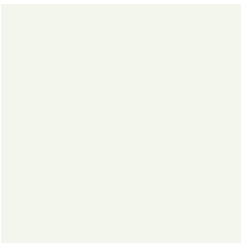
Number Sense and Operations

Patterns, Relations,
and Algebra

Geometry

MEASUREMENT

Data Analysis, Statistics,
and Probability



MATHEMATICS

STRAND: DATA ANALYSIS, STATISTICS, AND PROBABILITY

Prekindergarten

PK.DASP.1. Graph real objects or pictures of objects (no more than three) as a way to organize information (e.g., helps to make a graph [using actual shoes] showing how many children have sneakers with Velcro and how many have laces. Places cutouts of a hamster next to his favorite name for his new classroom pet).

PK.DASP.2. Describe and analyze information from graphs (e.g., says, "More kids like oranges than bananas" after looking at the tally marks next to the pictures of an orange and a banana. Says, "There are more boys than girls here today" after looking at the attendance graph.).

Kindergarten

K.DASP.1. Gather data about self and the environment to answer questions of interest to children; record the results using concrete graphs and simple picture graphs to display data.

K.DASP.2. Describe relationships displayed in graphs, tables, or other representations (e.g., Which has the most or least number of objects?).

Number Sense and Operations

Patterns, Relations,
and Algebra

Geometry

Measurement

DATA ANALYSIS,
STATISTICS, AND
PROBABILITY

Grade 1

1.DASP.1. Use surveys and observations to gather data about themselves and their surroundings (e.g., What is your favorite dessert?).

1.DASP.2. Represent and compare data (e.g., largest, smallest, most often, least often) using tally charts, pictures, and bar graphs.

1.DASP.3. Ask and answer simple questions related to data representations (e.g., Who is the tallest student in the class? What is the favorite fruit of the class?).

Grade 2

2.DASP.1. Use interviews, surveys, and observations to gather data about themselves and their surroundings.

2.DASP.2. Organize, classify, and represent data using tallies, charts, tables, bar graphs, pictographs, and Venn diagrams; interpret the representations.

2.DASP.3. Formulate inferences (draw conclusions) and make educated guesses (conjectures) about a situation based on information gained from data.

MATHEMATICS

STRAND: DATA ANALYSIS, STATISTICS, AND PROBABILITY, CONTINUED

Grade 3

3.DASP.1. Collect and organize data using observations, measurements, surveys, or experiments.

3.DASP.2. Construct, identify the main idea, and make predictions from various representations of data sets in the forms of tables, bar graphs (horizontal and vertical forms), pictographs, and tallies.

3.DASP.3. Record all possible outcomes for a simple event using concrete objects (e.g., tossing a coin).

3.DASP.4. Classify outcomes as certain, likely, unlikely, or impossible.

3.DASP.5. List and count the number of possible combinations of objects from 2 sets (e.g., How many different outfits can one make from a set of 2 sweaters and a set of 3 skirts?).

Grade 4

4.DASP.1. Collect and organize data using observations, measurements, surveys, or experiments, and identify appropriate ways to display the data.

4.DASP.2. Match a representation of a data set, such as lists, tables, or graphs (including circle graphs), with the actual set of data.

4.DASP.3. Compare two data sets represented in two bar graphs, pie graphs, and histograms.

4.DASP.4. Represent the possible outcomes for a simple probability situation (e.g., the probability of drawing a red marble from a bag containing 2 red marbles and 4 green marbles).

4.DASP.5. List and count the number of possible combinations of objects from 3 sets (e.g., How many different outfits can one make from a set of 3 shirts, a set of 2 skirts, and a set of 2 hats?).

Number Sense and Operations

Patterns, Relations,
and Algebra

Geometry

Measurement

DATA ANALYSIS,
STATISTICS, AND
PROBABILITY

Grade 5

- 5.DASP.1.** Define and apply the concepts of mean to solve problems.
- 5.DASP.2.** Construct, draw conclusions, and make predictions from various representations of data sets, including tables, line graphs, line plots, circle graphs, and bar graphs (where symbols or scales represent multiple units).
- 5.DASP.3.** Predict the probability of outcomes of simple experiments (e.g., tossing a coin, rolling a die) and test the predictions.

Grade 6

- 6.DASP.1.** Describe and compare data sets using the concepts of median, mean, mode, maximum and minimum, and range.
- 6.DASP.2.** Construct circle graphs using ratios, proportions, and percentages.
- 6.DASP.3.** Construct, label, and interpret stem-and-leaf plots.
- 6.DASP.4.** Use tree diagrams and other models (e.g., lists and tables) to represent possible or actual outcomes of trials.
- 6.DASP.5.** Represent two numerical variables on a scatterplot, and describe any apparent relationship that exists between the two variables (e.g., between time spent on homework and grades in class).
- 6.DASP.6.** Compute probabilities of events from simple experiments with equally likely outcomes (e.g., tossing dice, flipping coins, spinning spinners) by listing all possibilities and finding the fraction that meets given conditions. Analyze the outcomes.
- 6.DASP.7.** Use appropriate ratios between 0 and 1 to represent the probability of the outcome and associate the probability with the likelihood of the event; know that 0 probability means an event will not occur and that a probability of 1 means an event will occur.

MATHEMATICS

STRAND: DATA ANALYSIS, STATISTICS, AND PROBABILITY, CONTINUED

Grade 7

7.DASP.1. Find, describe, and interpret appropriate measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data.

7.DASP.2. Select, create, interpret, and use various tabular and graphical representations of data (e.g., circle graphs, Venn diagrams, stem-and-leaf plots, histograms, tables, and charts).

7.DASP.3. Describe the characteristics and limitations of a data sample. Identify different ways of selecting a sample (e.g., convenience sampling, responses to a survey, random sampling).

7.DASP.4. Use tree diagrams, tables, organized lists, and area models to compute probabilities for simple compound events (e.g., multiple coin tosses or rolls of dice).

7.DASP.5. Understand that the probability of either of two disjoint events occurring is the sum of the two individual probabilities and that the probability of one event following another, in independent trials, is the product of the two probabilities.

Grade 8

8.DASP.1. Revisit measures of central tendency (mean, median, and mode) and spread (range) that represent a set of data and then observe the change in each when an "outlier" is adjoined to the data set or removed from it. Use these notions to compare different sets of data and explain how each can be useful in a different way to summarize social phenomena such as price levels, clothing sizes, and athletic performances.

8.DASP.2. Select, create, interpret, and use various tabular and graphical representations of data (e.g., scatterplots, box-and-whisker plots).

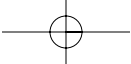
8.DASP.3. Recognize practices of collecting and displaying data that may bias the presentation or analysis.

8.DASP.4. Use data to estimate the probability of future events (e.g., batting averages).

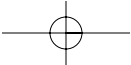
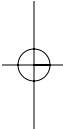
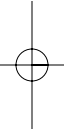
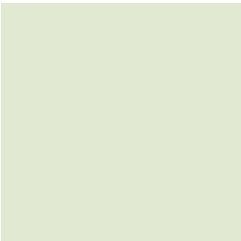
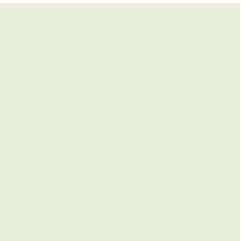
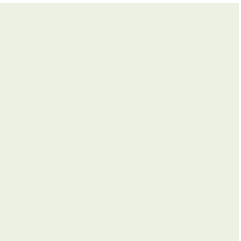
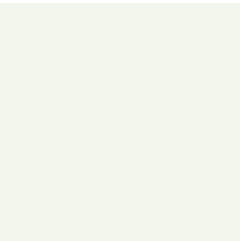
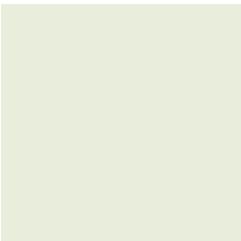
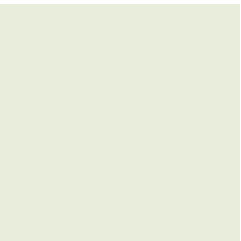
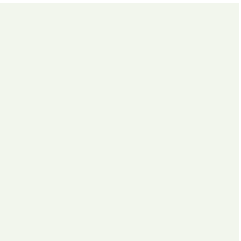
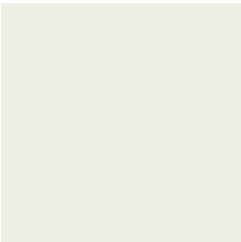
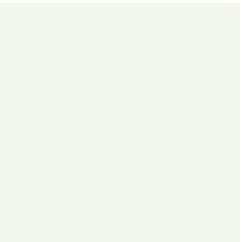
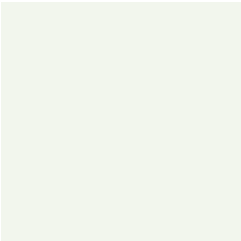
8.DASP.5. Select, create, interpret, and use various tabular and graphical representations of data; differentiate between continuous and discrete data and ways to represent them.

8.DASP.6. Apply the Fundamental Counting Principle (basic combinatorics) to find total number of outcomes possible for independent and dependent events, and calculate the probabilities using organized lists or tree diagrams.

8.DASP.7. Understand the difference between independent and dependent events, and recognize common misconceptions involving probability (e.g., Alice rolls a 6 on a die three times in a row; she is just as likely to roll a 6 on the fourth roll as she was on any previous roll).



| Number Sense and Operations | Patterns, Relations, and Algebra | Geometry | Measurement | DATA ANALYSIS, STATISTICS, AND PROBABILITY |
|-----------------------------|----------------------------------|----------|-------------|--|
|-----------------------------|----------------------------------|----------|-------------|--|



MATHEMATICS

STRAND: ALGEBRA I*

NUMBER SENSE AND OPERATIONS INDICATORS

AI.N.1. Use the properties of operations on real numbers, including the associative, commutative, identity, and distributive properties, and use them to simplify calculations.

AI.N.2. Simplify numerical expressions, including those involving integer exponents or the absolute value, e.g., $3(2^4 - 1) = 45$, $4|3 - 5| + 6 = 14$; apply such simplifications in the solution of problems.

AI.N.3. Calculate and apply ratios, proportions, rates, and percentages to solve a range of consumer and practical problems.

AI.N.4. Use estimation to judge the reasonableness of results of computations and of solutions to problems involving real numbers, including approximate error in measurement and the approximate value of square roots. (Reminder: This is without the use of calculators.)

AI.N.5. Understand the concept of n th roots of positive real numbers and of raising a positive real number to a fractional power. Use the rules of exponents also for fractional exponents.

AI.N.6. Apply the set operations of union and intersection and the concept of complement, universal set, and disjoint sets, and use them to solve problems, including those involving Venn diagrams.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS

AI.P.1. Recognize, describe, and extend patterns governed by a linear, quadratic, or exponential functional relationship or by a simple iterative process (e.g., the Fibonacci sequence).

AI.P.2. Use properties of the real number system to judge the validity of equations and inequalities and to justify every step in a sequential argument.

AI.P.3. Demonstrate an understanding of relations and functions. Identify the domain, range, and dependent and independent variables of functions.

AI.P.4. Translate between different representations of functions and relations: graphs, equations, sets of ordered pairs (scatter plots), verbal, and tabular.

AI.P.5. Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x - and y -intercepts from its graph or from a linear equation that represents the line.

AI.P.6. Find a linear function describing a line from a graph or a geometric description of the line (e.g., by using the point-slope or slope y -intercept formulas). Explain the significance of a positive, negative, zero, or undefined slope.

AI.P.7. Find linear functions that represent lines either perpendicular or parallel to a given line and through a point (e.g., by using the point-slope form of the equation).

AI.P.8. Add, subtract, and multiply polynomials with emphasis on 1st- and 2nd-degree polynomials.

* Standards A1.N.1–4 constitute a review of arithmetic material that should be familiar already from the earlier grades.

ALGEBRA I

Geometry

Algebra II

Probability and Statistics

Precalculus
and TrigonometryPATTERNS, RELATIONS, AND ALGEBRA INDICATORS *CONTINUED*

Al.P.9. Demonstrate facility in symbolic manipulation of polynomial and rational expressions by rearranging and collecting terms, factoring (e.g., $a^2 - b^2 = (a + b)(a - b)$, $x^2 + 10x + 21 = (x + 3)(x + 7)$, $5x^4 + 10x^3 - 5x^2 = 5x^2(x^2 + 2x - 1)$), identifying and canceling common factors in rational expressions, and applying the properties of positive integer exponents.

Al.P.10. Divide polynomials by monomials with emphasis on 1st- and 2nd-degree polynomials.

Al.P.11. Perform basic arithmetic operations with rational expressions and functions.

Al.P.12. Find solutions to quadratic equations (with real roots) by factoring, completing the square, or using the quadratic formula. Demonstrate an understanding of the equivalence of the methods.

Al.P.13. Solve equations and inequalities, including those involving absolute value of linear expressions (e.g., $|x - 2| > 5$), and apply to the solution of problems.

Al.P.14. Solve everyday problems (e.g., compound interest and direct and inverse variation problems) that can be modeled using linear or quadratic functions. Apply appropriate graphical or symbolic methods to the solution.

Al.P.15. Solve everyday problems (e.g., mixture, rate, and work problems) that can be modeled using systems of linear equations or inequalities. Apply algebraic and graphical methods to the solution.

DATA ANALYSIS, STATISTICS, AND PROBABILITY INDICATORS

Al.D.1. Select, create, and interpret an appropriate graphical representation (e.g., scatter plot, table, stem-and-leaf plots, circle graph, line graph, and line plot) for a set of data, and use appropriate statistics (e.g., mean, median, range, and mode) to communicate information about the data. Use these notions to compare different sets of data.

MATHEMATICS**STRAND: GEOMETRY**

- G.G.1.** Know correct geometric notation, including the notation for line segment \overline{AB} and angle $\angle ABC$.
- G.G.2.** Recognize special types of polygons (e.g., isosceles triangles, parallelograms, and rhombuses).
- G.G.3.** Apply properties of sides, diagonals, and angles in special polygons; identify their parts and special segments (e.g., altitudes, midsegments); determine interior angles for regular polygons.
- G.G.4.** Draw and label sets of points such as line segments, rays, and circles.
- G.G.5.** Detect symmetries of geometric figures.
- G.G.6.** Apply the triangle inequality and other inequalities associated with triangles (e.g., the longest side is opposite the greatest angle) to prove theorems and to solve problems.
- G.G.7.** Use properties and theorems about congruent and similar figures and about perpendicular and parallel lines to solve problems.
- G.G.8.** Write simple proofs of theorems in geometric situations, such as theorems about triangles, congruent and similar figures, and perpendicular and parallel lines (e.g., the longest side is opposite the greatest angle, two lines parallel to a third are parallel to each other; perpendicular bisectors of line segments are the set of all points equidistant from the two end points).
- G.G.9.** Distinguish between postulates and theorems. Use inductive and deductive reasoning, as well as proof by contradiction. Given a conditional statement, write its inverse, converse, and contrapositive.
- G.G.10.** Apply formulas for a rectangular coordinate system to justify theorems.
- G.G.11.** Draw congruent and similar figures using a compass, straightedge, or protractor. Justify the constructions by logical argument.
- G.G.12.** Apply congruence and similarity correspondences (e.g., $\triangle ABC \cong \triangle XYZ$) and properties of the figures to find missing parts of geometric figures, and provide logical justification.
- G.G.13.** Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.
- G.G.14.** Solve simple triangle problems using the triangle angle sum property and/or the Pythagorean theorem; study and understand more than one proof of this theorem.

G.G.15. Use the properties of special triangles (e.g., isosceles, equilateral, 30° - 60° - 90° , 45° - 45° - 90°) to solve problems.

G.G.16. Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

G.G.17. Demonstrate an understanding of the relationship between various representations of a line. Determine a line's slope and x - and y -intercepts from its graph or from a linear equation that represents the line. Find a linear equation describing a line from a graph or a geometric description of the line (e.g., by using the point-slope or slope y -intercept formulas). Explain the significance of a positive, negative, zero, or undefined slope.

G.G.18. Using rectangular coordinates, calculate midpoints of segments, slopes of lines and segments, and distances between two points, and apply the results to the solutions of problems.

G.G.19. Find linear equations that represent lines either perpendicular or parallel to a given line and through a point (e.g., by using the point-slope form of the equation).

G.G.20. Draw the results and interpret transformations on figures in the coordinate plane such as translations, reflections, rotations, scale factors, and the results of successive transformations. Apply transformations to the solution of problems.

G.G.21. Demonstrate the ability to visualize solid objects and recognize their projections, cross sections, and graph points in 3-D.

G.G.22. Find and use measures of perimeter, circumference, and area of common geometric figures such as parallelograms, trapezoids, circles, and triangles.

G.G.23. Find and use measures of lateral areas, surface areas, and volumes of prisms, pyramids, spheres, cylinders, and cones, and relate these measures to each other using formulas (e.g., find the volume of a sphere with a specified surface area).

G.G.24. Relate changes in the measurement (including units) of one attribute of an object to changes in other attributes (e.g., how changing the radius or height of a cylinder affects its surface area or volume).

G.G.25. Describe the effects of approximate error in measurement and rounding on measurements and on computed values from measurements.

G.G.26. Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense.

MATHEMATICS

STRAND: ALGEBRA II

NUMBER SENSE AND OPERATIONS INDICATORS

AII.N.1. Know and use the properties of operations on real numbers, including the existence of the identity and inverse elements for addition and multiplication and the existence of n th roots of positive real numbers for any positive integer n , and the n th power of positive real numbers.

AII.N.2. Simplify numerical expressions with powers and roots, including fractional and negative exponents.

AII.N.3. Know the representation of complex numbers (e.g., $a + bi$ where a and b are real numbers) and the procedures for adding, multiplying, and inverting complex numbers. Understand the associative, commutative, and identity properties for complex arithmetic.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS

AII.P.1. Describe, complete, extend, analyze, generalize, and create a wide variety of patterns, including iterative and recursive patterns such as Fibonacci Numbers and Pascal's Triangle.

AII.P.2. Identify arithmetic and geometric sequences and finite arithmetic and geometric series. Use the properties of such sequences and series to solve problems, including finding the formula for the general term and the sum, recursively and explicitly.

AII.P.3. Understand functional notation, evaluate a function at a specified point in its domain, and perform operations on functions with emphasis on the domain and range.

AII.P.4. Understand exponential and logarithmic functions and their basic arithmetic properties, including change of base and formulas for exponential of a sum and logarithm of a product.

AII.P.5. Given algebraic, numeric, and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential, and describe their behavior.

AII.P.6. Find solutions to radical equations; find solutions to quadratic equations (with real coefficients and real or complex roots) graphically, by factoring, by completing the square, or by using the quadratic formula.

AII.P.7. Solve a variety of equations and inequalities using algebraic, graphical, and numerical methods, including the quadratic formula. Include polynomial, exponential, and logarithmic functions, expressions involving the absolute values, and simple rational expressions.

AII.P.8. Explore matrices and their operations, including using them to solve systems of linear equations. Apply to solutions of everyday problems.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS *CONTINUED*

AII.P.9. Use symbolic, numeric, and graphical methods to solve systems of equations and/or inequalities involving algebraic, exponential, and logarithmic expressions. Describe the relationships among the methods.

AII.P.10. Solve everyday problems that can be modeled using polynomial, rational, exponential, logarithmic, and step functions; absolute values; and square roots. Apply appropriate graphical, tabular, or symbolic methods to the solution. Include compound interest, exponential growth and decay, and direct and inverse variation problems. .

AII.P.11. Recognize translations and scale changes of a given function $f(x)$ resulting from substitutions for the various parameters a , b , c , and d in $y = a f(b(x + c/b)) + d$. In particular, describe qualitatively the effect of such changes on polynomial, rational, exponential, and logarithmic functions.

AII.P.12. Simplify rational expressions. Solve rational equations and inequalities.

GEOMETRY INDICATORS

AII.G.1. Define the sine, cosine, and tangent of an acute angle. Apply to the solution of problems.

AII.G.2. Explain the identity $\sin^2\theta + \cos^2\theta = 1$. Relate the identity to the Pythagorean theorem.

AII.G.3. Relate geometric and algebraic representations of lines and simple curves.

DATA ANALYSIS, STATISTICS, AND PROBABILITY INDICATORS

AII.D.1. Select an appropriate graphical representation for a set of data and use appropriate statistics (e.g., quartile or percentile distribution) to communicate information about the data, including box plots.

AII.D.2. Use combinatorics (e.g., fundamental counting principle, permutations, and combinations) to solve problems, including computing geometric probabilities and probabilities of compound events.

MATHEMATICS

STRAND: PROBABILITY AND STATISTICS

PS.1. Demonstrate understanding of the definition of the notion of independent events and use the rules for addition, multiplication, and complementation to solve for probabilities of particular events in finite sample spaces.

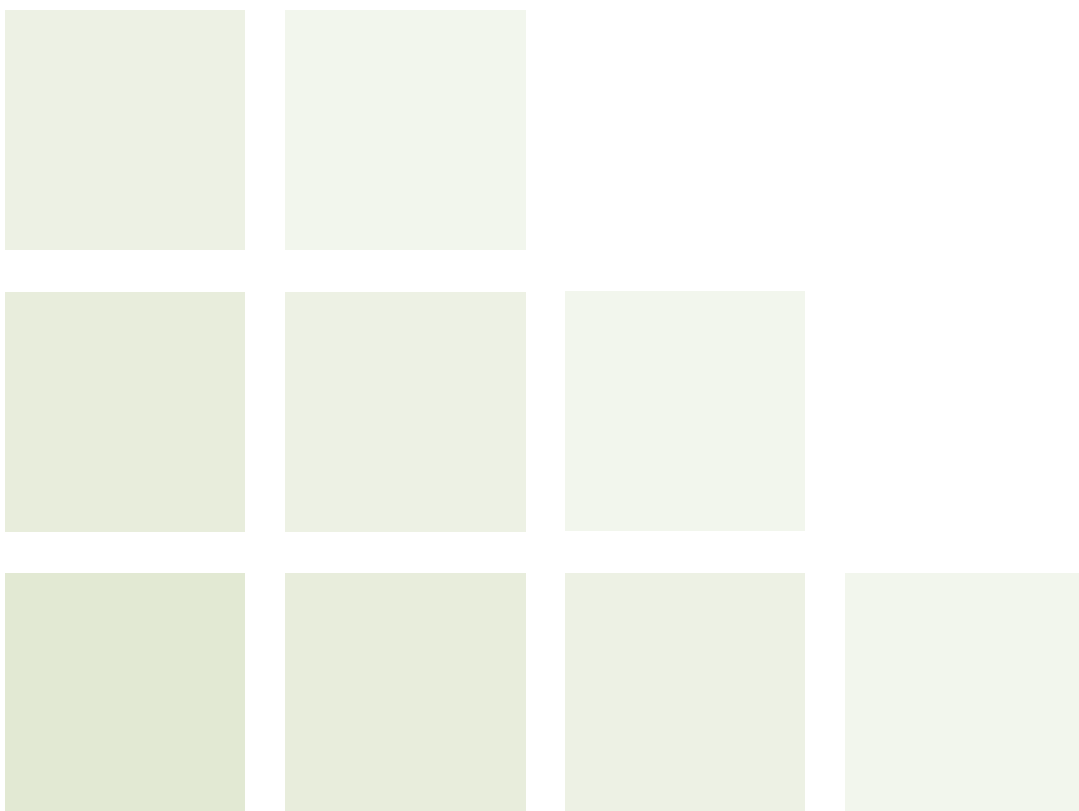
PS.2. Know the definition of conditional probability, and use it to solve for probabilities in finite sample spaces.

PS.3. Demonstrate understanding of the notion of discrete random variables by using them to solve for the probabilities of outcomes (e.g., the probability of the occurrences of five heads in 14 coin tosses).

PS.4. Apply uniform, normal, and binomial distributions to the solutions of problems.

PS.5. Determine the mean and the standard deviation of a normally distributed random variable.

PS.6. Know the definitions of the mean, median, and mode of a distribution of data, and compute each in particular situations.



Algebra I

Geometry

Algebra II

PROBABILITY AND STATISTICS

Precalculus
and Trigonometry

PS.7. Describe a set of frequency distribution data by spread (variance and standard deviation), skewness, symmetry, number of modes, or other characteristics. Use these concepts in everyday applications.

PS.8. Organize and describe distributions of data by using a number of different methods, including frequency tables, histograms, standard line and bar graphs, stem-and-leaf displays, scatter plots, and box-and-whisker plots.

PS.9. Describe and explain how the relative sizes of a sample and the population affect the validity of predictions from a set of data.

PS.10. Approximate a line of best fit (trend line) given a set of data (e.g., scatter plot).

MATHEMATICS

STRAND: PRECALCULUS AND TRIGONOMETRY

NUMBER SENSE AND OPERATIONS INDICATORS

PCT.N.1. Define and conduct operations on complex numbers, in particular, addition, subtraction, multiplication, and division. Relate the system of complex numbers to the systems of real and rational numbers.

PCT.N.2. Plot complex numbers using both rectangular and polar coordinates systems. Represent complex numbers using polar coordinates, i.e., $a + bi = r(\cos\theta + i\sin\theta)$.

PCT.N.3. Apply DeMoivre's theorem to multiply, take roots, and raise complex numbers to a power.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS

PCT.P.1. Relate the number of roots of a polynomial to its degree. Solve quadratic equations with complex coefficients, including use of completing the square.

PCT.P.2. Demonstrate an understanding of the trigonometric functions (sine, cosine, tangent, cosecant, secant, and cotangent). Relate the functions to their geometric definitions.

PCT.P.3. Use matrices to solve systems of linear equations. Apply to the solution of everyday problems.

PCT.P.4. Given algebraic, numeric, and/or graphical representations, recognize functions as polynomial, rational, logarithmic, or exponential.

PCT.P.5. Combine functions by composition, as well as by addition, subtraction, multiplication, and division.

PCT.P.6. Identify whether a function has an inverse and when functions are inverses of each other; explain why the graph of a function and its inverse are reflections of one another over the line $y = x$.

PCT.P.7. Identify maximum and minimum values of functions. Apply to the solution of problems.

PCT.P.8. Describe the translations and scale changes of a given function $f(x)$ resulting from substitutions for the various parameters a , b , c , and d in $y = a f(b(x + c/b)) + d$. In particular, describe the effect of such changes on polynomial, rational, exponential, and logarithmic functions.

PCT.P.9. Derive and apply basic trigonometric identities (e.g., $\sin^2\theta + \cos^2\theta = 1$, $\tan^2\theta + 1 = \sec^2\theta$) and the laws of sines and cosines.

PCT.P.10. Demonstrate an understanding of the formulas for the sine and cosine of the sum or the difference of two angles. Relate the formulas to DeMoivre's theorem and use them to prove other trigonometric identities. Apply to the solution of problems.

PCT.P.11. Understand, predict, and interpret the effects of the parameters a , ω , b , and c on the graph of $y = a\sin(\omega(x - b)) + c$; do the same for the cosine and tangent. Use to model periodic processes.

PCT.P.12. Translate among geometric, algebraic, and parametric representations of curves. Apply to the solution of problems.

PATTERNS, RELATIONS, AND ALGEBRA INDICATORS *CONTINUED*

PCT.P.13. Relate the slope of a tangent line at a specific point on a curve to the instantaneous rate of change. Explain the significance of a horizontal tangent line. Apply these concepts to the solution of problems.

PCT.P.14. Approximate areas under a curve.

PCT.P.15. Demonstrate an understanding of the binomial theorem and use it in the solution of problems.

PCT.P.16. Identify maximum and minimum values of functions in simple situations. Apply to the solution of problems.

GEOMETRY INDICATORS

PCT.G.1. Demonstrate an understanding of the laws of sines and cosines. Use the laws to solve for the unknown sides or angles in triangles. Determine the area of a triangle given the length of two adjacent sides and the measure of the included angle.

PCT.G.2. Use vectors to solve problems. Describe addition of vectors, multiplication of a vector by a scalar, and the dot product of two vectors, both symbolically and geometrically. Use vector methods to obtain geometric results.

PCT.G.3. Apply properties of angles, parallel lines, arcs, radii, chords, tangents, and secants to solve problems.

MEASUREMENT INDICATORS

PCT.M.1. Describe the relationship between degree and radian measures, and use radian measure in the solution of problems, particularly problems involving angular velocity and acceleration.

PCT.M.2. Use dimensional analysis for unit conversion and to confirm that expressions and equations make sense.

DATA ANALYSIS, STATISTICS, AND PROBABILITY INDICATORS

PCT.D.1. Design surveys and apply random sampling techniques to avoid bias in the data collection.

PCT.D.2. Apply regression results and curve fitting to make predictions from data and select appropriate functions as models.

PCT.D.3. Compare the results of simulations (e.g., random number tables, random functions, and area models) with predicted probabilities.

MATHEMATICS

GLOSSARY

Absolute Value A number's distance from zero on the number line. The absolute value of -4 is 4; the absolute value of 4 is 4.

Addend A number that is to be added.

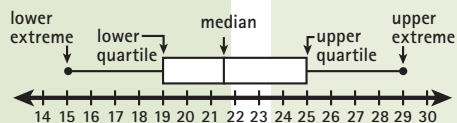
Algorithm A finite set of steps for completing a procedure (e.g., long division).

Analog Having to do with data represented by continuous variables (e.g., a clock with hour, minute, and second hands).

Associative Property Giving the same result irrespective of the order taken; thus, addition and multiplication are associative, but subtraction and division are not. For addition, $(a + b) + c = a + (b + c)$. For multiplication, $(a \times b) \times c = a \times (b \times c)$.

Binomial Theorem A mathematical formula used to calculate the value of a two-part mathematical expression that is squared, cubed, or raised to another power or exponent without explicitly multiplying the parts themselves.

Box-and-Whisker Plot A method for displaying the median, quartiles, and extremes of a set of data using the number line.



Calculus The mathematics of change and motion. The main concepts of calculus are limits, instantaneous rates of change, and areas enclosed by curves.

Capacity The maximum amount or number that can be contained or accommodated (e.g., a jug with a one-gallon capacity; the auditorium was filled to capacity).

Cartesian Plane See Coordinate Plane.

Closure Property A set of numbers, such as the integers, is closed under a particular operation if performing the operation on numbers in the set results in another number in that set. For example, the set of nonzero integers is closed under multiplication but is not closed under division because the product of two nonzero integers is again a nonzero integer, but their quotient need not be an integer.

Commutative Properties Giving the same result in mathematics or logic irrespective of the order in which two or more terms or quantities are placed. For example, $a + b = b + a$.

Complex Number A number that can be written in the form $a + bi$ where a and b are real numbers and $i = \sqrt{-1}$.

Composite Numbers A natural number that is not prime.

Congruent Two figures in the plane or in space are congruent if one can be transformed into the other by a sequence of translations and rotations.

Convenience (Haphazard) Sampling The collection of data from readily available sources without emphasis on methodological rigor.

Coordinate Plane A plane in which two coordinate axes are specified, i.e., two intersecting directed straight lines, usually perpendicular to each other, and usually called the x -axis and y -axis. Every point in a coordinate plane can be described uniquely by an ordered pair of numbers, the coordinates of the point with respect to the coordinate axes.

Decimal Number Any real number expressed in base 10 notation, such as 2.673.

Deductive Reasoning A type of reasoning wherein the conclusion about particulars follows necessarily from general or universal premises.

Digit Any of the Arabic numerals 0 to 9.

Digital Having to do with data that is represented in the form of numerical digits; providing a readout in numerical digits (e.g., a digital watch).

Dilation A type of transformation of the plane or of the space that fixes a point C (the center of the dilation) and maps any other point P to the point P' characterized as follows: the segment CP has the same direction as the segment CP' , and has length k times the length of the segment CP , where k is a positive constant (the scale factor of the dilation).

Divisibility Rules Patterns that allow you to determine whether or not a number divides evenly into another number (leaving no remainder) without actually doing the division (e.g., a number that ends in 5 or 0 is divisible by 5).

Dot Product For vectors $A = \langle x_a, y_a \rangle$ and $B = \langle x_b, y_b \rangle$, the dot product $A \bullet B = (x_a)(x_b) + (y_a)(y_b)$.

Equilateral Used to describe a geometric figure in which all sides are of equal length.

Expanded Notation A way of representing a number that shows the sum of each digit multiplied by the appropriate positive power of ten and the units digit (e.g., 3,451 as $3 \times 1,000 + 4 \times 100 + 5 \times 10 + 1$ or as $3 \times 10^3 + 4 \times 10^2 + 5 \times 10 + 1$).

Exponent The number that indicates how many times the base is used as a factor (e.g., in $4^3 = 4 \times 4 \times 4 = 64$, the exponent is 3, indicating that 4 is repeated as a factor three times).

Fact Family A set of arithmetic identities that use the same three numbers. For example, the numbers 1, 13, and 14 can be used to make two addition problems, $1 + 13 = 14$ and $13 + 1 = 14$, and two subtraction problems, $14 - 13 = 1$ and $14 - 1 = 13$.

Factors Any of the two or more quantities that are multiplied together. In the expression 3.712×11.315 , the factors are 3.712 and 11.315. The factors of 6 are 2 and 3.

Fibonacci Numbers The sequence of numbers beginning with 1, 1, in which each number that follows is the sum of the previous two numbers, i.e., 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 ...

Function A mathematical relation that associates each object in a set with exactly one value.

Fundamental Counting Principle If event M can occur in m ways, and, after it has occurred, event N can occur in n ways, then event M followed by event N can occur $m \cdot n$ ways.

Geometric Pattern A sequence of symbols or geometric figures.

Geometric Sequence (Progression) An ordered list of numbers that has a common ratio between consecutive terms, e.g., $1/2, 1/4, 1/8, \dots$.

Greatest Common Factor The greatest common factor of two numbers, a and b , is the largest number that divides both a and b evenly.

Histogram A vertical block graph with no spaces between the blocks. It is used to represent frequency data in statistics.

Identity Properties For addition, $a + 0 = a$. For multiplication, $a \times 1 = a$.

Inductive Reasoning (a) The type of reasoning that uses inference to reach a generalized conclusion from particular instances; (b) In mathematics, demonstration of the validity of a law concerning all the positive integers by proving that it holds for the integer 1 and that if it holds for an arbitrarily chosen positive integer k it must hold for the integer $k + 1$; also called mathematical induction.

Integers The set consisting of the natural numbers, their negatives, and zero.

MATHEMATICS

Irrational Number A number that cannot be expressed as a quotient of two integers (e.g., $\sqrt{2}$). It can be shown that a number is irrational if and only if it cannot be written as a repeating or terminating decimal.

Isosceles Triangle A triangle in which two of the three sides are of equal length.

Iterative Pattern or Sequence A sequence or pattern formed by repeating the same procedure (for example, the Fibonacci sequence).

Least Common Multiple The least common multiple of two numbers, a and b , is the smallest number that contains both a and b as factors.

Line Graph A set of data points on an X-Y grid with consecutive points connected by line segments.

Line of Best Fit Intuitively a straight line drawn through or near to as many data points as possible on a scatter plot.

Line Plot A number line with dots or other marks above it to show the number of times a number or event occurs in a set of data.

Linear Equation In two variables (x, y), any equation that can be written in the form $Ax + By + C = 0$, where A and B cannot both be 0. The graph of such an equation is a line.

Manipulatives Various objects or tools used to teach abstract concepts through hands-on activities.

Matrix (pl. Matrices) A rectangular array of numbers or variables.

Mean In statistics, the average obtained by dividing the sum of two or more quantities by the number of these quantities.

Measure of Central Tendency Either the mean, median, or mode of a distribution (i.e., one of the possible notions of an average).

Median In statistics, the quantity designating the middle value in a set of numbers.

Mixed Number A number that is written as a whole number and a fraction.

Mode In statistics, the value that occurs most frequently in a given series of numbers.

Monomial In the variables x, y, z , a monomial is an expression of the form $ax^m y^n z^k$ in which m, n , and k are nonnegative integers and a is a constant (e.g., $5x^2$, $3x^2y$, or $7x^3yz^2$).

Nets An arrangement of two-dimensional figures that can be folded to make a polyhedron (a solid figure with flat faces that are polygons).

Nonstandard Unit Unit of measurement expressed in term of objects (such as paper clips, sticks of gum, shoes, etc.).

Numeral A symbol or mark used to represent a number.

Numeric Pattern A sequence, array, or other arrangement of numbers according to some fixed rule, e.g., Fibonacci sequence, Pascal's triangle.

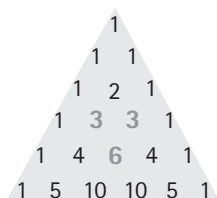
Order of Operations (1) Do all of the operations inside parentheses and/or above and below a fraction bar in the proper order; (2) Find the value of any powers or roots; (3) Multiply and divide from left to right; (4) Add and subtract from left to right.

Ordinal Number A number designating the place (as first, second, or third) occupied by an item in an ordered sequence.

Parallel Given distinct lines in a plane that are infinite in both directions, the lines are parallel if they never meet. The distinct lines in the coordinate plane are parallel if and only if they have the same slope.

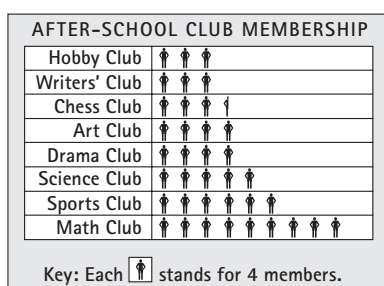
Partitioning A process of dividing an object into parts.

Pascal's Triangle A triangular arrangement of numbers in which each row starts and ends with 1 and each other number is the sum of the two numbers above it.



Perpendicular At right angles to a line or plane.

Pictogram (Pictograph) A graph that uses pictures to show and compare information.



Polynomial In algebra, a sum of monomials; for example, $x^2 + 2xy + y^2$.

Prime Factorization A number written as the product of all its prime factors.

Prime Number A natural number greater than one that can only be divided without a remainder by itself and one. The first seven primes are 2, 3, 5, 7, 11, 13, 17.

Probability A number between zero and one that describes the likelihood that a given event

will take place. For example, the probability of throwing a six with a single throw of one die is $1/6$ and the probability of throwing two sixes with a single throw of two dice is $1/36$.

Proof A line of argument based on deductive reasoning.

Proportion See Ratio.

Pythagorean Theorem For any right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.

Quadratic Function A polynomial in one variable of degree 2 (e.g., $y = ax^2 + bx + c$).

Quadrilateral A two-dimensional geometric figure with four sides.

Random Sampling A smaller group of people or objects chosen from a larger group or population by a process giving equal chance of selection to all possible people or objects.

Random Variable A variable that is itself a function of the result of a statistical experiment in which each outcome has a definite probability of occurrence; also called a *variate*.

Range In statistics, the difference between the smallest and the largest values in a frequency distribution.

Ratio A comparison of two numbers or quantities, e.g., 4 to 7 or $4 : 7$ or $4/7$.

Rational Number A number that can be written as the ratio of an integer to a counting number or, more formally, a number that can be expressed as a ratio a/b where a and b are integers and $b > 0$ (e.g., 0.5, $3/5$, -3 , 8, $3, 9/10$).

Real Number Any number on the continuous number line. The real numbers are divided into rational and irrational numbers.

Recursive Pattern or Sequence A pattern or sequence wherein each successive term can be computed from some or all of the preceding terms by an algorithmic procedure.

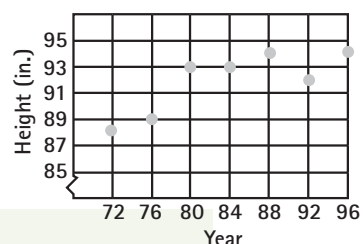
Reflection A type of transformation that flips points about a line, called the *line of reflection*. Taken together, the image and the pre-image have the line of reflection as a line of symmetry.

Rotation A type of transformation that turns a figure about a fixed point, called the *center of rotation*.

Sample Space In probability, the set of all outcomes of a given experiment, e.g., the sample space for tossing two coins is (H,H), (H,T), (T,H), (T,T).

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Scatter Plot Data plotted as ordered pairs in the coordinate plane.



Scientific Notation A widely used floating-point system in which numbers are expressed as products consisting of a number between 1 and 10 multiplied by an appropriate power of 10 (e.g., $562 = 5.62 \times 10^2$).

Sequence A set of elements ordered so that they can be labeled with consecutive positive integers starting with 1 (e.g., 1, 3, 9, 27, 81). In this sequence, 1 is the first term, 3 is the second term, 9 is the third term, and so on).

Similarity In geometry, two shapes R and S are similar if there is a dilation D (see the definition of dilation) that takes S to a shape congruent to R. It follows that R and S are similar if they are congruent after one of them is expanded or shrunk.

Square Root In real arithmetic, the square root of a nonnegative number x is that nonnegative number y for which $y^2 = x$. In complex arithmetic, a square root (note the indefinite article) of x is any number y for which $y^2 = x$. **Example:** In real arithmetic, $\sqrt{16} = 4$ and $\sqrt{-16}$ is not defined. In complex arithmetic, the square roots of 16 are 4 and -4 , and the square roots of -16 are $4i$ and $-4i$.

Symmetry A symmetry of a shape S in the plane or space is a rigid motion T that takes S onto itself ($T(S) = S$). For example, reflection through a diagonal and a rotation through a right angle about the center are both symmetries of the square.

Transformation A prescription or rule that sets up a one-to-one correspondence between the points in a geometric object (the pre-image) and the points in another geometric object (the image). Reflections, rotations, translations, and dilations are particular examples of transformations.

Translation A type of transformation that moves every point by the same distance in the same direction.

Tree Diagram A diagram of a hierarchical structure that shows the relationships between components as branches.

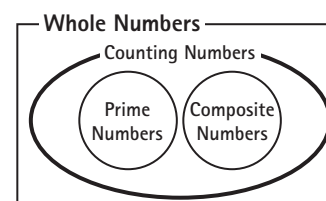
Turn Around Facts The commutative property of addition (e.g., $2 + 5 = 7$, $5 + 2 = 7$).

Valid Well grounded or justifiable (e.g., a valid theory); logically correct.

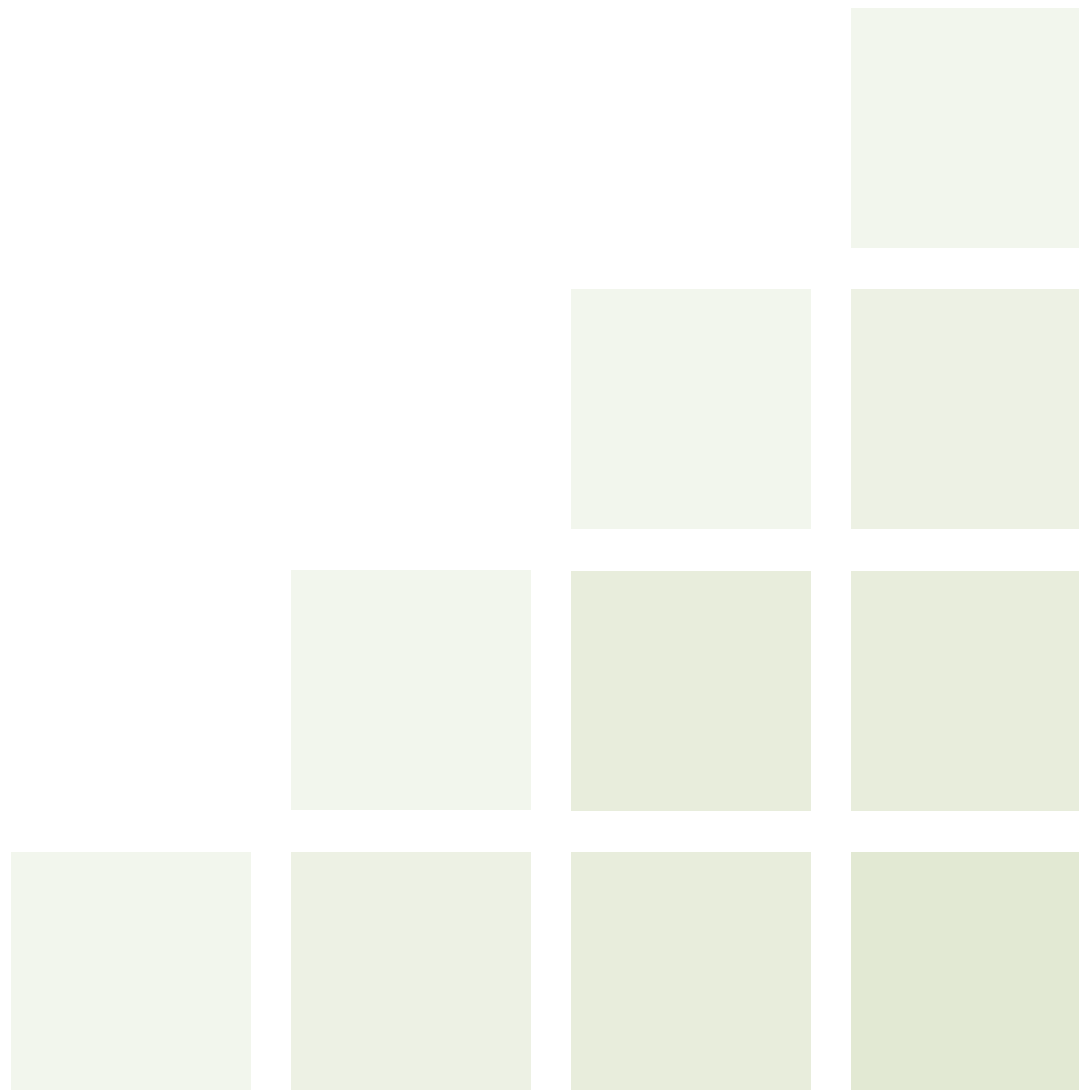
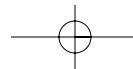
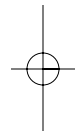
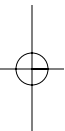
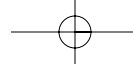
Variable A letter used to represent one or more numbers in an expression, equation, inequality, or matrix.

Vector A quantity that has magnitude and direction. A vector is typically represented by a directed line segment whose length represents the magnitude and whose orientation in space represents the direction.

Venn Diagram A diagram used to show relationships between sets.



Whole Number A number that is either a counting number or zero.



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ACKNOWLEDGMENTS

Working under the direction of the DCPS Office of Academic Services and StandardsWork, Inc., a DC-based nonprofit that specializes in helping school districts develop world-class learning standards, the following educators, parents, and community members helped develop the learning standards in reading/English language arts and mathematics for the District of Columbia. Their contributions are greatly appreciated.

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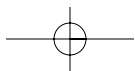
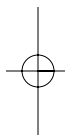
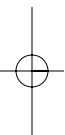
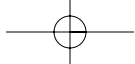
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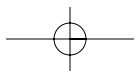
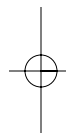
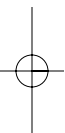
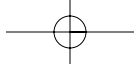
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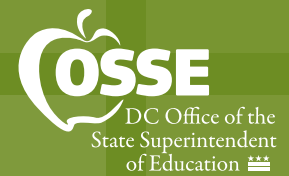


RELATED RESOURCES ALSO AVAILABLE

Grade-level parent guides to the standards, translated into several languages, are available on the OSSE Web site at <http://osse.dc.gov>.

The Office of Academic Services at D.C. Public Schools also has developed grade-level curriculum guides, with sample learning activities, year at a glance references, unit roadmaps, standards-based worksheets, sample assessment items, and other resources to help teachers, administrators, students, parents, and community members better understand the new learning standards.

These and other materials are available on the DCPS Web site at www.k12.dc.us.



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